

Stability of Grid-Connected Inverter with LCL Filter

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Abstract — The paper analyses oscillations in a single-phase grid connected inverter with the LCL output filter. Passive and active damping techniques are designed and compared by simulations. The inverter is controlled by a proportional resonant controller.

Keywords — control, dc/ac converter, filtering

I. INTRODUCTION

Grid connected pulse width modulated (PWM) voltage source converters require an inductive filter for its operations. These converters are used in many applications such as the active power filters, PWM rectifiers or PWM inverters. To decrease the weight and volume of the filter it is desirable to use a higher-order filter. Usually an LCL filter is used [2–14]. The LCL filter has a drawback of possible resonance if excited at a resonant frequency. This resonance introduces the system instability and increases THD of current. To suppress the oscillations a damping technique is used. There are two ways of the oscillation damping in the LCL filter: the passive and active damping. Both of them have its advantages and disadvantages and are still analysed and designed. The passive damping [1] uses damping resistors which introduces additional system losses. An active damping is used to remove these extra losses but keep system stability. The active damping of the LCL filter is studied in the active power filters [2–4], PWM rectifiers [5–7], PWM inverters [8–12] and in general grid interacting converters [13–15]. This paper analyses various damping techniques for the single-phase grid connected inverter with the LCL filter. Several damping techniques are analysed and designed for a single phase inverter with the proportional-resonant (PR) controller.

II. LCL FILTER

The grid-connected voltage source inverter (VSI) needs an inductive load. This load is provided mainly by the output filter. The filter topology depends on the harmonics attenuation requirements. The harmonics are produced by the pulse-width modulation (PWM). Usually higher order filters, such as the LCL filter (Fig. 1), are used. The LCL filter is a third order filter with attenuation of 60 dB/dec. The advantage of the high attenuation of a third order system has a drawback of resonance. The LCL filter has three resonant frequencies defined by the reactive components. The grid influences the resonant frequency of the LCL filter as well. Connecting the LCL filter to the output of a PWM modulated inverter causes driving the LCL filter input with a spectrum of various high-frequency voltages.

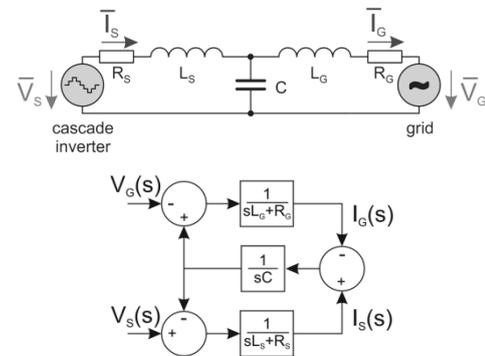


Fig. 1. LCL filter topology and its dynamical model (R_s and R_G are parasitic resistances).

The inverter output voltage V_s with a frequency equal to a resonant frequency of the LCL filter defined by (1) causes a resonance in the output current I_G of the LCL filter. The resonance causes system instability and increases the THD of the grid current and thus it is undesirable.

$$f_{0I_G V_s} = \frac{1}{2\pi} \sqrt{\frac{L_s + L_G}{L_s L_G C}} \quad (1)$$

The example of the harmonic spectrum of the LCL filter output current is shown in Fig. 2. The resonance peak is clearly visible and its contribution to the THD of the grid current I_G is more significant than the contribution from the inverter switching frequency.

Suppression of the switching frequency is guaranteed by a proper design of the LCL filter reactive components. Suppression of the LCL filter resonance is guaranteed by a proper damping technique design. However, as it is shown, some damping techniques influence the harmonics suppression.

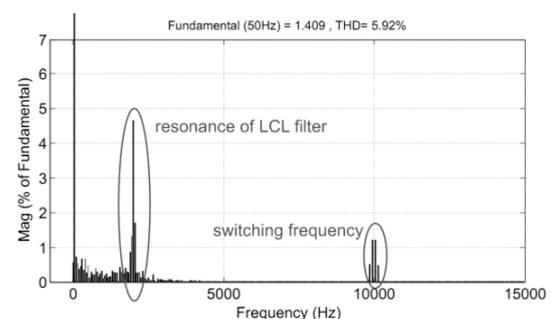


Fig. 2. Example of LCL filter output current (with rms value 1.409 A) spectrum without proper oscillation damping.

The attenuation of the switching frequency in the voltage V_s with respect to the grid current I_G is described by the transfer function (2).

$$\frac{I_G(s)}{V_s(s)} = \frac{1}{s^3 L_S L_G C + s^2 (L_S C R_G + R_S C L_G) + s (R_S C R_G + L_S + L_G) + R_S + R_G} \quad (2)$$

III. RESONANCE DAMPING

There are two ways how to damp the LCL filter. The so-called passive damping employs an extra resistor added in series with a filter capacitor. This resistor influences the transfer function (2) and suppresses the resonance by modifying the transfer function for frequencies around and above the resonant frequency defined by (1).

The frequency characteristics of the system can be changed also by modifying the controller. The proper transfer function of the controller can damp the LCL filter oscillations. The main idea is to remove undesired frequencies from the inverter output voltage by modifying the modulation signal of the PWM. This approach is called an active damping.

A. Passive Damping

The passive damping is ensured by adding a dissipative component to the LCL filter topology. The properly placed resistor will decrease the resonance peak. The damping resistor is usually placed in series with the filter capacitor (Fig. 3).

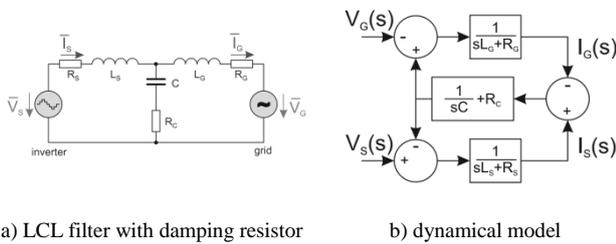


Fig. 3. LCL filter with passive damping resistor.

The damping resistor will change the transfer function of the LCL filter (3).

$$\frac{I_G(s)}{V_s(s)} = \frac{1 + sCR_C}{s^3 L_S L_G C + s^2 (L_S C R_G + R_S C L_G + R_C C L_G + L_S C R_C) + s (R_S C R_G + L_S + L_G + R_S R_C C + R_C C R_G) + R_S + R_G} \quad (3)$$

The damping resistor adds losses to the LCL filter. The value of the resistor is usually chosen as one third of the capacitor reactance at the resonant frequency:

$$R_C = \frac{1}{3} \frac{1}{2\pi f_{0L} V_S C} \quad (4)$$

The advantage of the passive damping is its robustness. However there are also disadvantages. The damping resistor has losses which decrease the overall system efficiency. The losses on the damping resistor R_C connected in series with the capacitor can be calculated [16] where h represents harmonic order:

$$P_{R_C} = R_C \sum_h [i_s(h) - i_G(h)]^2 \quad (5)$$

The second main disadvantage is decrease of the higher frequency damping (Fig. 4). If the resonant

frequency of the LCL filter is not too far from the switching frequency, this decrease is minimal.

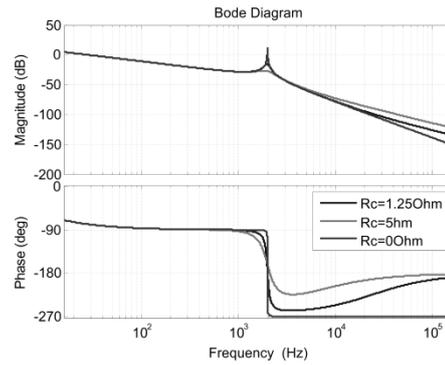


Fig. 4. LCL with various damping resistor values.

B. Active Damping

The frequency characteristics of the system can be changed also by modifying the controller. The proper transfer function of the controller can damp the LCL filter oscillations. The main idea is to remove the undesired frequencies from the inverter output voltage by modifying the PWM modulation signal. This approach is called the active damping.

C. Active Damping – Notch Filter

There are several ways how to achieve the active damping. Probably the most straightforward one is to remove the resonant frequency from the inverter output voltage by a filter (Fig. 5).

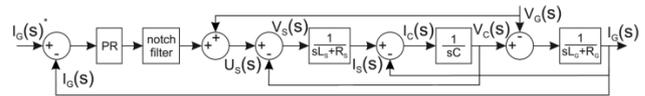


Fig. 5. Simplified control structure of PR current control with active damping by notch filter.

The suitable type of the filter is a notch (negative peak) filter. The simple notch filter consists of the LC resonant tank (Fig. 6).

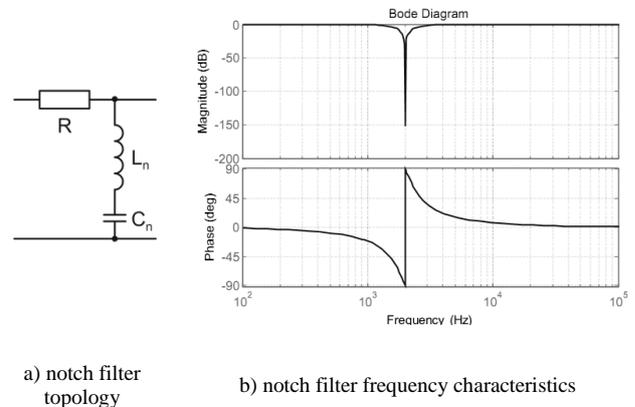


Fig. 6. Notch filter.

The transfer function of a notch filter is defined by (6).

$$F_{notch}(s) = \frac{L_n C_n s^2 + 1}{L_n C_n s^2 + R_n C_n s + 1} \quad (6)$$

where:

$$L_n = 1mH \quad (7)$$

$$C_n = \frac{1}{4\pi^2 L_n f_{0IGVS}^2} \quad (8)$$

$$R_n = \sqrt{\frac{(2\pi f_n)^2 L_n C_n - 1}{(2\pi f_n C_n)^2}} \quad (9)$$

The frequency f_n defines the notch filter bandwidth around the LCL filter resonant frequency f_{0IGVS} .

The advantage of the active damping by the notch filter is its sensorless concept. When compared to the passive damping, the system transfer function for higher frequencies is not influenced by inserting the notch filter. However, the design of the notch filter depends on the LCL filter parameters.

D. Active Damping – Virtual Resistance

Another active damping method uses the concept of a virtual resistor. The virtual resistor method is based on the LCL filter capacitor current sensing and multiplying of this current by the virtual damping resistor resistance. The resulting virtual voltage is then subtracted from the inverter PWM modulating voltage (Fig. 7).

The virtual resistor value can be calculated using (2). As can be seen from Fig. 8, the virtual resistor does not change the system frequency characteristic for higher frequencies than the f_{0IGVS} resonant frequency.

The transfer function of the LCL filter damped by virtual damping resistor R_C is changed to:

$$\frac{I_G(s)}{V_S(s)} = \frac{1}{s^3 L_S L_G C + s^2 (L_S C R_G + R_S C L_G + R_C C L_G) + s (R_C R_G C + R_S R_G C + L_S + L_G) + R_S + R_G} \quad (10)$$

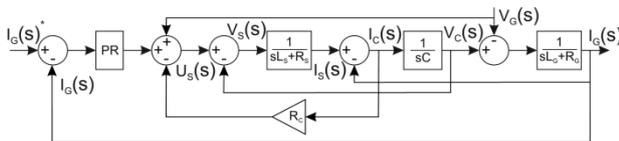


Fig. 7. Simplified control structure of PR current control with active damping by virtual resistor.

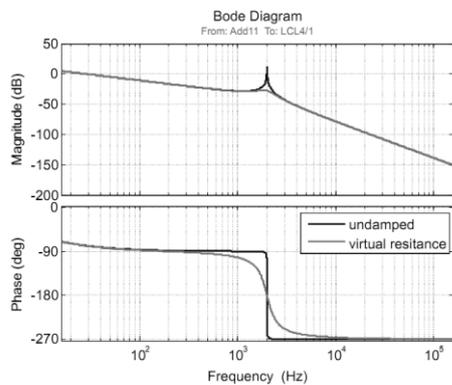


Fig. 8. Frequency characteristics of LCL filter with active damping by virtual resistor.

IV. CONTROL SYSTEM DESIGN

A. LCL Filter Parameters

The performance of three damping methods is compared by simulation. The system consists of a single-phase PWM voltage source inverter with switching frequency of 5 kHz and dc link voltage of 420 V. The inverter is connected to the grid through the LCL filter with parameters shown in Table I.

TABLE I. LCL FILTER PARAMETERS

LCL filter parameter	Value	Unit
Apparent power S	1.2	kW
Switching frequency f_{sw}	5	kHz
Current ripple of I_S	20	%
Resonant frequency f_{0IGVS}	2	kHz
Inductance L_S	3.6	mH
Inductance L_G	1.8	mH
Capacitance C	5.3	μF

B. Proportional Resonant Controller

The inverter is a single-phase system. It is thus beneficial to use the proportional resonant (PR) controller (Fig. 9). This approach will remove the need to transform the single-phase system into the rotating reference frame in dq coordinates for the PI controller.

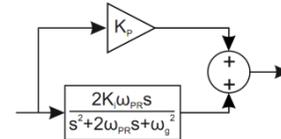


Fig. 9. PR controller.

The PR controller has a transfer function (11).

$$F_{PR}(s) = K_P + \frac{2K_I \omega_{PR} s}{s^2 + 2\omega_{PR} s + \omega_g^2} \quad (11)$$

The frequency ω_{PR} defines the PR controller bandwidth around the grid frequency ω_g . To design the PR controller it is necessary to know the transfer function from the inverter voltage V_S (manipulated variable) to the grid current I_G (controlled variable). The grid voltage V_G is a measured disturbance and is compensated in the PR controller. The grid current is a measured controlled variable. The high frequency transfer function from V_S to I_G of the LCL filter is:

$$\frac{I_G(s)}{V_S(s)} = \frac{1}{s^3 L_S L_G C + s^2 (L_S C R_G + R_S C L_G) + s (R_S C R_G + L_S + L_G) + R_S + R_G} \quad (12)$$

The PR controller controls the grid current I_G with the grid frequency and thus generates the manipulated variable V_S with the grid frequency. The transfer function (3) is simplified by omitting the high-frequency terms:

$$\left. \frac{I_G(s)}{V_S(s)} \right|_{LL} = \frac{1}{s(L_S + L_G) + R_S + R_G} \quad (13)$$

The LCL filter is therefore simplified to a first order system with the time constant of:

$$T_{LCL} = \frac{L_S + L_G}{R_S + R_G} \quad (14)$$

And its gain:

$$K_{LCL} = \frac{1}{R_S + R_G} \quad (15)$$

The PR controller is designed to compensate the time constant T_{LCL} . The proportional gain of the PR controller is set to (τ is the time constant of the required control dynamics of the whole controlled system):

$$K_P = \frac{T_{LCL}}{\tau K_{LCL}} \quad (16)$$

The integral gain of the PR controller is set to:

$$K_I = \frac{K_P}{T_{LCL}} \quad (17)$$

TABLE II. PR CONTROLLER PARAMETERS

PR Controller Parameters	Value	Unit
Time constant T_{LCL}	13.5	Ms
Gain K_{LCL}	2.5	-
Proportional gain K_P	5.4	-
Integral gain K_I	400	-
Frequency ω_{PR}	1	rad/s
Grid frequency ω_G	314	rad/s

V. COMPARISON OF DAMPING METHODS

The stability of the system created by PR controller and LCL filter is checked by gain and phase margins obtained from Bode characteristics. The system is considered stable if the gain margin is at least 10 dB and the phase margin is at least 60 deg . Besides the control loop, the damping of the LCL filter frequency characteristic is compared as well.

The frequency characteristics of the open control loop with designed PR controller and undamped LCL filter is shown in Fig. 10. It has a gain margin of -46.7 dB and phase margin of -89.7 deg , and is clearly unstable. The negative peak in phase at 315 rad/s is caused by resonant frequency of PR controller.

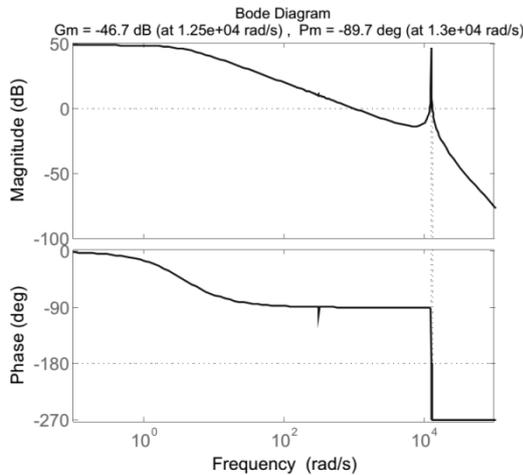


Fig. 10. Frequency characteristics of open control loop with PR controller and undamped LCL filter.

A. Passive Damping Method

The damping resistor R_C for the LCL filter specified in Table I calculated using (2) is 5Ω . Figure 11 shows frequency characteristics of the undamped LCL filter and LCL filter damped by passive damping resistor.

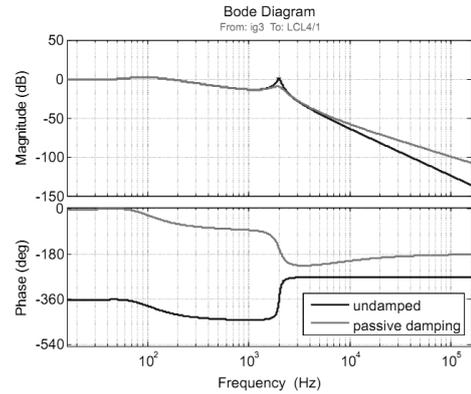


Fig. 11. LCL filter frequency characteristics with passive damping.

The designed passive damping causes a positive gain margin of 13.4 dB and a positive phase margin of 90.2 deg . (Fig. 12). The stability of the system is thus ensured.

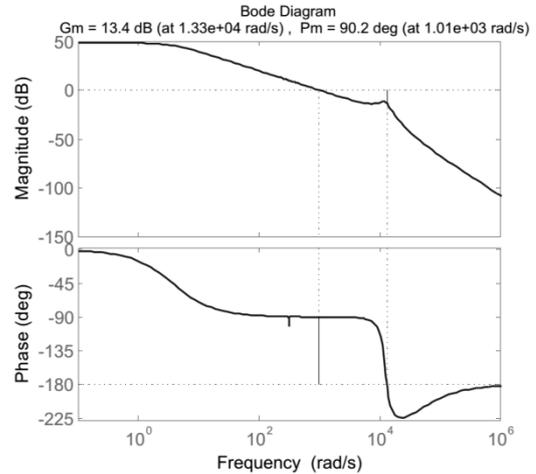


Fig. 12. Frequency characteristics of open control loop with PR controller and LCL filter with passive damping.

Figure 13 shows the dynamic response of the grid current. The stability is clearly visible in the time domain as well without influencing the system dynamic behaviour.

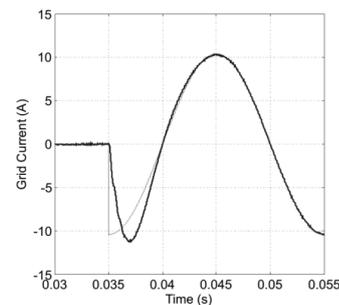


Fig. 13. LCL filter output current and its reference signal with passive damping.

B. Notch Filter

The simulated notch filter transfer function is:

$$F_{notch}(s) = \frac{6.36 \cdot 10^{-9} s^2 + 1}{6.36 \cdot 10^{-9} s^2 + 4.664 \cdot 10^{-5} s + 1} \quad (18)$$

To suppress the resonant frequency the notch filter has a narrow bandwidth of 1 kHz. Figure 14 shows the frequency characteristics of the undamped filter and the filter damped with the notch filter. The resonant peak is completely removed.

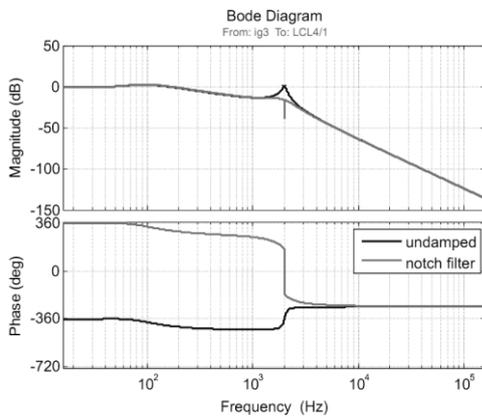


Fig. 14. LCL filter frequency characteristics with notch filter damping.

The notch filter damping (Fig. 5) causes a positive gain margin of 17.2 dB and a positive phase margin of 87.5 deg (Fig. 15).

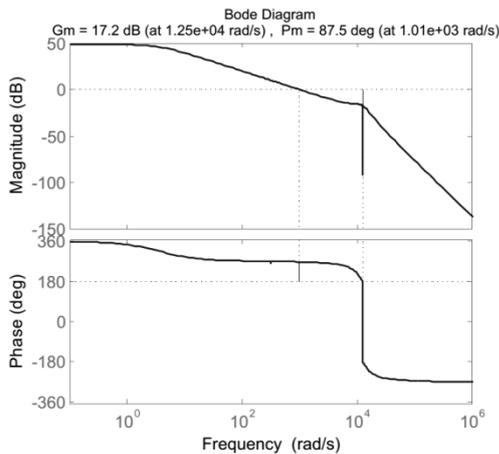


Fig. 15. Frequency characteristics of open control loop with PR controller and LCL filter damped by notch filter.

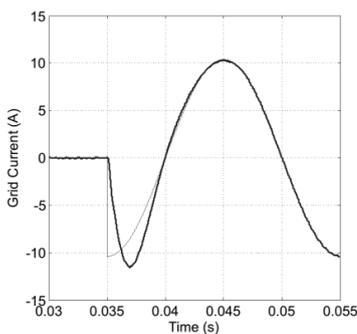


Fig. 16. LCL filter output current and its reference signal with notch filter damping.

C. Virtual Resistance

To create a damping in the controlled system a virtual damping resistor with resistance of 10 Ω (two times the passive damping resistor) was chosen. Figure 17 shows the frequency characteristics of the undamped LCL filter and the LCL filter damped with the virtual resistor.

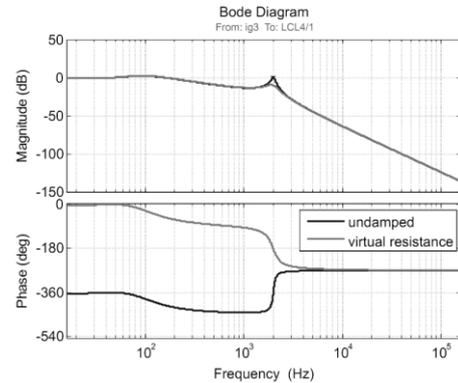


Fig. 17. LCL filter frequency characteristics with virtual resistance damping.

After checking the stability of the virtual damping method only a 8.89 dB gain margin was observed (Fig. 18). The system is stable but it is advisable to use a three times the value of calculated passive damping resistor (4) to ensure a gain margin at least 10 dB.

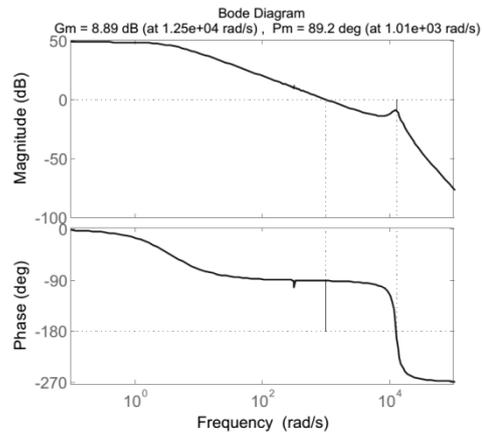


Fig. 18. Frequency characteristics of open control loop with PR controller and LCL filter damped with virtual resistor.

The smaller gain margin caused by the proposed virtual damping resistor is also visible in Fig. 19 as damped oscillations during a transient period.

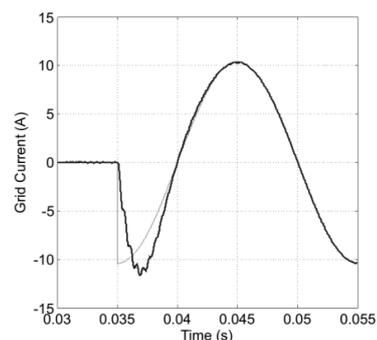


Fig. 19. LCL filter output current and its reference signal with passive damping virtual resistance damping.

VI. CONCLUSIONS

The paper describes the oscillation phenomena in the grid-connected inverter with the LCL filter. The LCL filter resonance problem is analysed in the single-phase inverter with the PWM control. The PR controller is used to control the grid current. The controller is designed and verified by simulation. It is shown that the PR controller by its own cannot ensure the stability of the system. Thus three different techniques are analysed and designed to suppress the LCL filter oscillations. The passive damping technique has the advantage of robustness and simplicity. Unfortunately the passive damping increases the system losses and the LCL filter capabilities of harmonics damping are reduced. The two active damping techniques remove these disadvantages. The notch filter at the output of the controller can be realised in a digital form. Thus no additional hardware costs are included. The notch filter needs to be tuned for a particular filter and its resonant frequency and is thus less robust. The additional current sensor is required for the virtual resistor damping technique. The capacitor current sensing will increase the robustness. The simple passive damping technique can be used to calculate the value of a virtual damping resistor. Choosing of a suitable damping technique depends on an application and usually an active damping together with passive damping is used.

ACKNOWLEDGMENT

The authors wish to thank the project VEGA 1/0464/15 for its support.

The work was supported by project FEI-2015-3.

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