A 6DOF Motion Platform with Permanent Magnet Linear Motors

Evžen Thöndel
Department of Electric Drives and Traction, Czech Technical University in Prague, Technická 2, 166 27 Prague, Czech Republic, e-mail: *thondee@fel.cvut.cz*

Abstract — **The paper deals with the modelling and simulation of a six-degree-of-freedom (6DOF) motion platform with permanent magnet linear actuators. The notion and structure of a 6DOF platform (a.k.a. Stewart platform) is well known. The main aim of this paper is to find and describe a suitable solution for applications requiring high dynamics and position accuracy. For this reason, permanent magnet linear actuators, characterized by their high dynamics and accuracy, have been used – despite their still considerable (albeit gradually decreasing) price tag per unit of power. Furthermore this paper develops a mathematical model to be later employed for the scaling and design of the platform controls.**

Keywords — Motion platform, linear actuator, permanent magnet synchronous motor, kinematic transformation, control.

I. Introduction

Motion platforms with six degrees of freedom were originally designed for use in aircraft simulators [5]. However, the system soon found many applications in other industries, such as automated production or testing, the latter area being also the main motivation for this study.

The platform is comprised of six linear actuators arranged in a parallel kinematic structure. Different platform types employ different linear actuators, with the oldest ones featuring hydraulic cylinders. Gradually, hydraulic solutions started to be replaced by electromechanical actuators [6]. However, another approach is imminent, following the recent breakthroughs in the area of permanent magnet electric drives, as purely electric linear actuators now start to offer a fully worked out alternative to the hydraulic or electromechanical solutions. The main benefit of electric actuators is a much higher dynamic range, allowing systems based on this method to be used in a wider variety of applications.

Wind tunnel analyses and measurements are performed by all airborne research organisations. A new motion platform, intended for aerodynamic process measurements and determination of dynamic parameters, is currently being developed by the CIDAM (Centre for Intelligent Drives and Advanced Machine Control) competence centre with the support of the Technology Agency of the Czech Republic (TACR). With the new platform researchers will be able to execute certain limited manoeuvres and test the aircraft control algorithms directly in a wind tunnel. More details of the plans using the platform during wind tunnel testing are available in [3].

The following paragraphs provide a description of the process how the mathematical model of the 6DOF motion platform with permanent magnet linear actuators was developed. In doing so, great attention was paid in particular to the planned purpose of the system, i.e. kinematic range and actuator output sizing as well as control design. In addition, simulation for a given set of parameters is described at the end of this paper.

A schematic drawing of the mechanism described in this paper is provided in Figure 1.

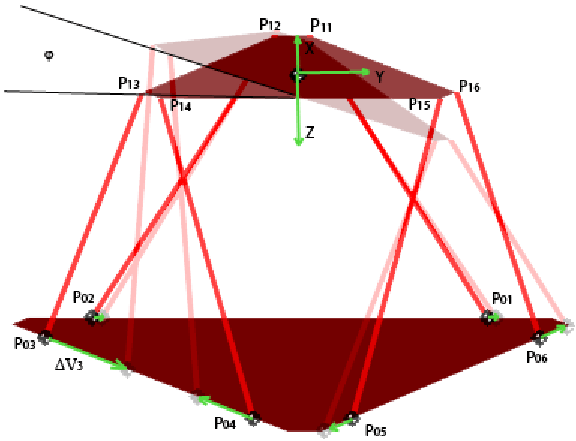

Fig. 1: Motion Platform with Six Degrees of Freedom Driven by Linear Actuators

Given the properties and requirements of permanent magnet linear motors a different linear actuator design was used in this project. Unlike hydraulic or electromechanical systems with flexible actuator arm lengths, the permanent magnet electric actuators used in this motion platform feature fixed-length arms. To reach the required position the arm bottom joint slides along a rail fitted with permanent magnets.

II. Inverse Kinematic Transformation

In order to describe the relationships of the inverse kinematic transformation (i.e. transformation from the position of the platform movable frame to the positions of individual actuators), let us first define the following symbols and expressions:

- O_1 is the coordinate system of the movable frame, which is capable to move freely (with six degrees of freedom) in relation to the fixed frame.
- O_2 is the coordinate system of the fixed frame.
- O_α is the coordinate system of an actuator, defined by means of the vectors u, v, w (see below).

P^1_{1j} is the actuator joint in the movable frame, expressed in the coordinates of the movable frame, $j \in \langle 1 \cdots 6 \rangle$.

P^0_{0j} is the actuator joint in the fixed frame, expressed in the coordinates of the fixed frame, $j \in \langle 1 \cdots 6 \rangle$.

ΔV_j is the displacement of the linear motor, $j \in \langle 1 \cdots 6 \rangle$..

T is the transformation matrix of points in the coordinate system of the movable frame to the coordinate system of the fixed frame.

$$T = \begin{bmatrix} c\psi c\vartheta & c\psi s\vartheta s\varphi - s\psi c\varphi & c\psi s\vartheta c\varphi + s\psi s\varphi & \Delta x \\ s\psi c\vartheta & s\psi s\vartheta s\varphi + c\psi c\varphi & s\psi s\vartheta c\varphi - c\psi s\varphi & \Delta y \\ -s\vartheta & c\vartheta s\varphi & c\vartheta c\varphi & \Delta z \\ & 0_{1x3} & & 1 \end{bmatrix}$$

ψ, ϑ, φ are the rotation angles of the movable frame along the Z, Y and X axes in relation to the fixed frame. The same order is used also for the transformation matrix.

Δx, Δy, Δz Is the displacement of the movable frame in relation to the fixed frame.

Basic kinematic parameters:

r_1, r_0 Is the radius of the circumscribed circle of the movable (r_1) and fixed (r_0) frames.

f_1, f_0 Is the distance of the joints at the edge of the movable (f_1) and fixed (f_0) frames.

L Is the length of the actuator arm.

Using the above parameters, the points P^1_{11} and P^1_{12} can be defined by the following expressions:

$$P^1_{11} = \begin{bmatrix} -r_1 + \frac{\sqrt{3}}{2} f_1 \\ \frac{f_1}{2} \\ 0 \end{bmatrix}, P^1_{12} = \begin{bmatrix} -r_1 + \frac{\sqrt{3}}{2} f_1 \\ \frac{-f_1}{2} \\ 0 \end{bmatrix} \quad (1)$$

The remaining points, i.e. $P^1_{1j}, j \in \langle 3 \ldots 6 \rangle$, can be defined by means of a +120 ° or -120 ° rotation, i.e. multiplication by the following rotation matrix:

$$R_{120°} = \begin{bmatrix} \frac{1}{2} & \frac{-\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{-1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}, R_{-120°} = R^{-1}_{120°} = R^T_{120°} \quad (2)$$

The very same approach can be used also when defining the points of the fixed frame, i.e. $P^0_{0j}, j \in \langle 1 \cdots 6 \rangle$.

If the position of the moving frame in relation to the fixed frame is known the following expression holds true (assuming the points are given in homogenous coordinates = addition of a fourth coordinate of 1):

$$P^0_{1j} = TP^1_{1j} \quad (3)$$

Let us now define the coordinate system of the actuator, where the X axis runs in the direction of the linear motor motion and can be defined (for actuator 1) by the vector:

$$\overrightarrow{u_1} = \frac{P^0_{02} - P^0_{01}}{|P^0_{02} - P^0_{01}|} \quad (4)$$

The Y axis is perpendicular to the X axis and the actuator arm is located in the XY plane. The following expression holds true for the vector in the direction of the actuator arm:

$$\overrightarrow{v_1^{**}} = P^0_{11} - P^0_{01} = TP^1_{11} - P^0_{01} \quad (5)$$

The vector perpendicular to X can be determined by means of orthogonalization:

$$\overrightarrow{v_1^{*}} = \overrightarrow{v_1^{**}} + k\overrightarrow{u_1} \quad (6)$$

$$\overrightarrow{u_1}\overrightarrow{v_1^{*}} = 0 \rightarrow k = -\overrightarrow{u_1}\overrightarrow{v_1^{**}} \quad (7)$$

As the last step the orthogonal vector has to be normalised:

$$\overrightarrow{v_1} = \frac{\overrightarrow{v_1^{*}}}{\left|\overrightarrow{v_1^{*}}\right|} \quad (8)$$

The Z axis is perpendicular to the XY plane. The vector defining this axis can be obtained by means of the vector product:

$$\overrightarrow{w_1} = \overrightarrow{u_1} \times \overrightarrow{v_1} \quad (9)$$

The transformation matrix from the coordinate system of the actuator to the coordinate system of the fixed frame can be expressed as:

$$T_\alpha = \begin{bmatrix} [\overrightarrow{u_1} \quad \overrightarrow{v_1} \quad \overrightarrow{w_1}]_{R_\alpha} & P^0_{01} \\ 0_{1x3} & 1 \end{bmatrix} \quad (10)$$

The inverse transformation can be written by inverting the foregoing transformation matrix:

$$T_\alpha^{-1} = \begin{bmatrix} R^T_\alpha & -R^T_\alpha P^0_{01} \\ 0_{1x3} & 1 \end{bmatrix} \quad (11)$$

A vector diagram of the whole calculation is provided in Figure 2 below.

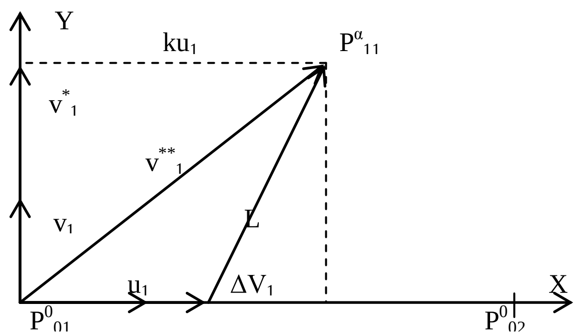

Fig. 2: Vector analysis of the linear motor displacement calculation

Using the above transformations, the displacement of the linear motor with respect to a given position of the movable frame can be written as:

$$P^\alpha_{11} = T_\alpha^{-1} TP^1_{11} \quad (12)$$

$$\Delta V_1 = P^\alpha_{11}(x) - \sqrt{L^2 - P^\alpha_{11}(y)} \quad (13)$$

III. Permanent Magnet Synchronous Linear Motor

Permanent magnet synchronous motors (PMSMs) belong to the most recent motor generations, finding use especially in applications requiring accurate position and speed servo-control (such as industrial robots). Unlike other electric motors, PMSMs can be easily arranged in a linear shape. This feature will be utilized in the development of the mathematical model.

The mathematical model of a linear motor can be derived from the properties of typical rotational motors – all what one has to do is to "cut and unroll" the engine in one's mind [4]. All properties of the mathematical model of a rotational motor can be recalculated to a linear form if the radius of the "unrolled" rotational motor is known. The radius can be written as:

$$2\pi r = 2p\tau \rightarrow r = \frac{\tau p}{\pi}, \tag{14}$$

where r is the desired radius, p the number of pole pairs and τ the pole pitch.

Rotational electric machines are typically analysed in a suitable coordinate system rotating synchronously with the selected quantity. In this way, the examined AC quantities can be transformed to the corresponding DC quantities. A system rotating with a synchronous speed seems to be best suited for synchronous machines. In technical literature, this transformation is known as the d-q transform, or Park's transformation.

The currents can be transformed to the d-q system by means of a gradual α-β transformation (Clarke's transformation) to the stationary orthogonal system:

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & \frac{-1}{2} & \frac{-1}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{-\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \tag{15}$$

and subsequently to the rotating d-q system:

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} cos\vartheta_e & sin\vartheta_e \\ -sin\vartheta_e & cos\vartheta_e \end{bmatrix}\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} \tag{16}$$

The mathematical model has been determined under the following assumptions:

- The system is powered by a three-phase symmetrical power source with harmonic voltages.
- All phases have the same resistances and inductances.
- The magnetization characteristic is linear.
- Iron losses are not considered.

The following expressions are applied to the different stator winding phases:

$$u_a = R_s i_a + \frac{d}{dt}\psi_a \tag{17}$$

$$u_b = R_s i_b + \frac{d}{dt}\psi_b \tag{18}$$

$$u_c = R_s i_c + \frac{d}{dt}\psi_c \tag{19}$$

where R_s is the stator winding resistance and ψ_a, ψ_b and ψ_c are the magnetic fluxes in the form:

$$\psi_a = L_s i_a + \psi_M cos(\omega_e t) \tag{20}$$

$$\psi_b = L_s i_b + \psi_M cos\left(\omega_e t + \frac{2}{3}\pi\right) \tag{21}$$

$$\psi_c = L_s i_c + \psi_M cos\left(\omega_e t - \frac{2}{3}\pi\right) \tag{22}$$

In the above equation, L_s is the stator winding inductance and ψ_M the rotor magnetic flux, rotating with respect to the stator with the speed ω_e.

The final equations of the electrical part of the motor can be obtained by means of a d-q transformation of the above formulas:

$$u_d = R_s i_d + L_s \frac{d}{dt} i_d - L_s \omega_e i_q \tag{23}$$

$$u_q = R_s i_q + L_s \frac{d}{dt} i_q + L_s \omega_e i_d + \psi_M \omega_e \tag{24}$$

The foregoing set of equations can be completed by the equation of the electromechanical torque, which can be derived using the law of conservation of energy, assuming mechanical losses and iron losses are not considered.

$$\frac{3}{2} Re\{U_s I_s\} = M\omega_m + 3R_s I_s^2, \omega_m = \frac{\omega_e}{p} \tag{25}$$

$$M = \frac{3}{2}\psi_M p i_q \tag{26}$$

Now, the radius expression determined earlier can be used and the above model "unrolled" into linear form.

$$\omega_e = p\omega_m = p\frac{v}{r} = \frac{\pi}{\tau} v \tag{27}$$

$$M = Fr = \frac{\tau p}{\pi} F \tag{28}$$

The linear motor equations can be obtained by substituting into the rotation motor equations (23), (24) and (26):

$$u_d = R_s i_d + L_s \frac{d}{dt} i_d - L_s \frac{\pi}{\tau} v i_q \tag{29}$$

$$u_q = R_s i_q + L_s \frac{d}{dt} i_q + L_s \frac{\pi}{\tau} v i_d + \psi_M \frac{\pi}{\tau} v \tag{30}$$

$$F = \frac{3}{2}\frac{\pi}{\tau}\psi_M i_q = K_F i_q \tag{31}$$

where KF is the force constant, v the motor mechanical speed ($\Delta\dot{V}_j$) and F the acting force.

In the derived equations, the current component i_d generates a magnetic flux inverse to the magnetic flux generated by the permanent magnets. In practice, the value of this current component is maintained as zero by means of independent vector control. Assuming that $i_d = 0$, the foregoing equations can be further simplified without any impact on the model suitability for the design and testing of the motion platform. Following this step, the resulting synchronous linear motor differential equation can be written as:

$$\frac{d}{dt} i_q = -\frac{R_s}{L_s} i_q + \frac{1}{L_s} u_q - \frac{2}{3}\frac{K_F}{L_s} v \tag{32}$$

$$F = K_F i_q \tag{33}$$

IV. Mathematical Model of the Parallel Mechanism

The analytical expression of the dynamic behaviour of the parallel mechanism is highly complex, as the equation of motion cannot be obtained without forward kinematic transformation, an exceedingly complicated operation for this mechanism type. In fact, no solution of this problem has been found yet for the Stewart platform [2].

The equation of motion has the following general form [6]:

$$m_{red}\Delta\ddot{V}_j + \frac{1}{2}\frac{\partial m_{red}}{\partial \Delta V_j}\Delta\dot{V}_j^2 = F - m_{red} g - B\Delta\dot{V}_j \tag{34}$$

In this expression, F is the acting force of the linear motor according to expression (33), ΔV_j the displacement of the linear motor according to expression (13), B the viscous friction coefficient, g the gravitational acceleration and m_{red} the reduced mass of the given actuator load. Generally, this mass depends on the position of the platform and therefore cannot be determined without forward kinematic transformation.

$$m_{red} = f(\Delta V_1, \dots \Delta V_6) \tag{35}$$

A mathematical model was created in the MATLAB/ Simulink/SimMechanics environment for simulation purposes to provide numerical solutions of the forward kinematic transformation (Figure 3).

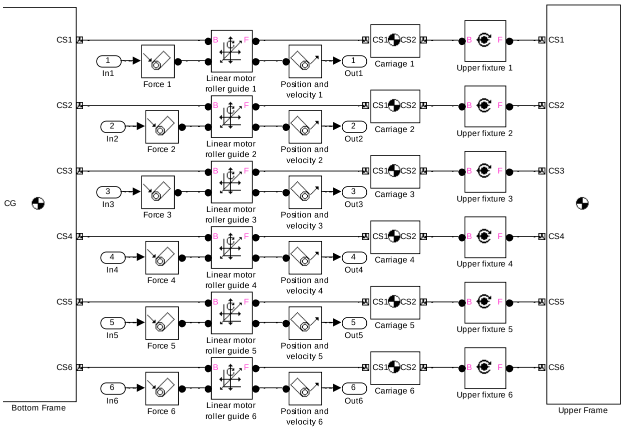

Fig. 3: Mathematical Model of the Mechanism in the MATLAB/Simulink/SimMechanics Environment.

In addition, the simulation model is planned to be used also for the control algorithm design and testing. For this purpose, the model can be linearized, assuming that the total load mass m_{load} is divided approximately evenly between individual actuators. In such a scenario, the following expression holds true:

$$m_{red} = const = \frac{m_{load}}{6} \tag{36}$$

Under these simplified conditions, the equation of motion for an actuator can be expressed in the following linear differential matrix form, which is suitable for control algorithm designs and analyses. The control algorithm must be robust enough with respect to the m_{red} variation expressed by formula (35).

$$\frac{d}{dt}\begin{bmatrix} i_q \\ v \\ \Delta V_j \end{bmatrix} = \begin{bmatrix} -\frac{R_S}{L_S} & -\frac{2}{3}\frac{K_F}{L_S} & 0 \\ \frac{K_F}{m_{red}} & -\frac{B}{m_{red}} & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} i_q \\ v \\ \Delta V_j \end{bmatrix} + \begin{bmatrix} \frac{1}{L_S} & 0 \\ 0 & -1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} u_q \\ g \end{bmatrix} \tag{37}$$

V. Control System

As the most frequent drive control method, standard cascading PI control (current, speed, position) has been used for the simulations described below – see Figure 4. Various newer control methods could, in theory, be used and analysed as well, but the associated control quality improvements remain questionable in the light of the increased computing requirements and control system price tag. A more detailed analysis of the synchronous motor control impacts featuring modern control methods such as MPC (Model Predictive Control) is provided in [1]; the study shows, both in simulations and practical experiments, that the results are virtually the same for both methods. Modern control methods may be much more useful in scenarios where different types of drive constraints have to be considered, which are difficult to implement by means of the cascade PI control.

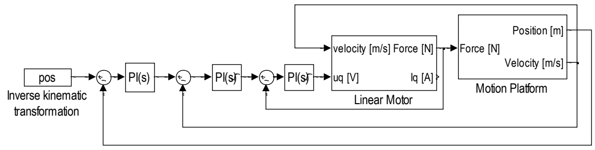

Fig. 4: Control System

VI. Simulation Results

The derived simulation model was implemented in the MATLAB/Simulink environment. A general overview of the parameters used in the simulation is provided in Table 1 below.

TABLE I.
MODEL PARAMETERS USED DURING SIMULATION

Parameter	Value	Unit
R_s	5.7	Ω
L_s	40	mH
K_F	145.5	N/A
m_{load}	150	kg
B*	0	$kg.s^{-1}$
r_0	0.5	m
r_1	0.5	m
f_0	0.05	m
f_1	0.05	m

* Mechanical losses were not taken into consideration in the initial simulation.

The sinusoidal shapes of the platform position curves (i.e. the positions of the movable frame in relation to the fixed frame) were used as the model inputs. A summary overview of the simulation results is provided in Figure 6 below.

So far, it has not been possible to verify the simulation results by actual measurements, as the platform does not exist yet. As explained at the beginning of this paper, the purpose of the mathematical and simulation model is to lay the foundations for the subsequent design and sizing of the mechanism and its control system. The parameters of the linear motor were taken over from the manufacturer's catalogue [8]. The curves in the simulation chart correspond with the expected values.

An online visualisation of the simulation is available via YouTube at [7].

The main motivation for the development of a motion platform with linear motors is the overall increase of dynamics, thereby widening the application spectrum to the field of dynamic testing. Fig. 5 compares the measured dynamic characteristics of the previous solutions based on hydraulic and electromechanical actuators with the expected (simulated) dynamic characteristics of the new solution based on linear motors. All compared types of actuators have similar power characteristics. As shown in the figure the new solution has much higher (20x) dynamics in contrast to the previous solutions.

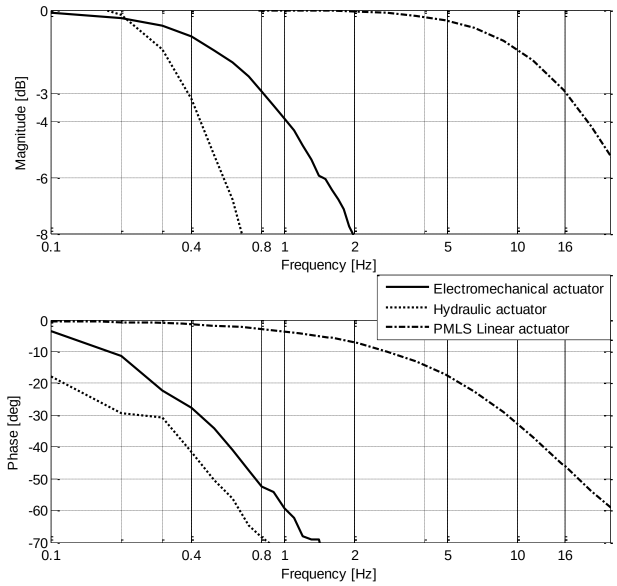

Fig. 5: Frequency characteristics

VII. CONCLUSION

The paper provides a description of the modelling and simulation of a 6DOF motion platform actuated by permanent magnet linear motors. It introduces a detailed mathematical model and presents the results of initial simulations for a given set of parameters. The model described here will be employed to design an actual mechanism to be later used for aerodynamic parameter measurements in a wind tunnel.

REFERENCES

[1] K. Belda, "Mathematical Modelling and Predictive Control of Permanent Magnet Synchronous Motor Drives", Transaction on Electrical Engineering, Vol. 2, No. 4, 2013.

[2] J. Domagoj and L. Budin, "Forward Kinematics of a Stewart Platform Mechanism", 6th International Conference on Inteligent Engineering Systems, May 2002.

[3] P. Hospodář, "Stewartova platform". Research study of Aerospace Research and Test Establishment, Prague, Czech Republic, unpublished.

[4] CE. Miller, AW. Zyl and CF. Landy, "Modelling a Permanent Magnet Linear Synchronous Motor For Control Purposes", IEEE Africon, 2002.

[5] D. Stewart, "A Platform with Six Degrees of Freedom", UK Institution of Mechanical Engineers Proceedings, Vol 180, 1965.

[6] E. Thöndel, "Design and Optimal Control of a Linear Electromechanical Actuator for Motion Platforms with Six Degrees of Freedom", Intelligent Automation and Systems Engineering, Springer, 65-77, 2011.

[7] E. Thöndel, "Simulation of a 6DOF Motion Platform with Linear Permanent Magnet Synchronous Motors", Online: http://youtu.be/AZjgcrV642c, 2014.

[8] Raveo, s.r.o., "Lineární motory", Online: http://www.raveo.cz/.

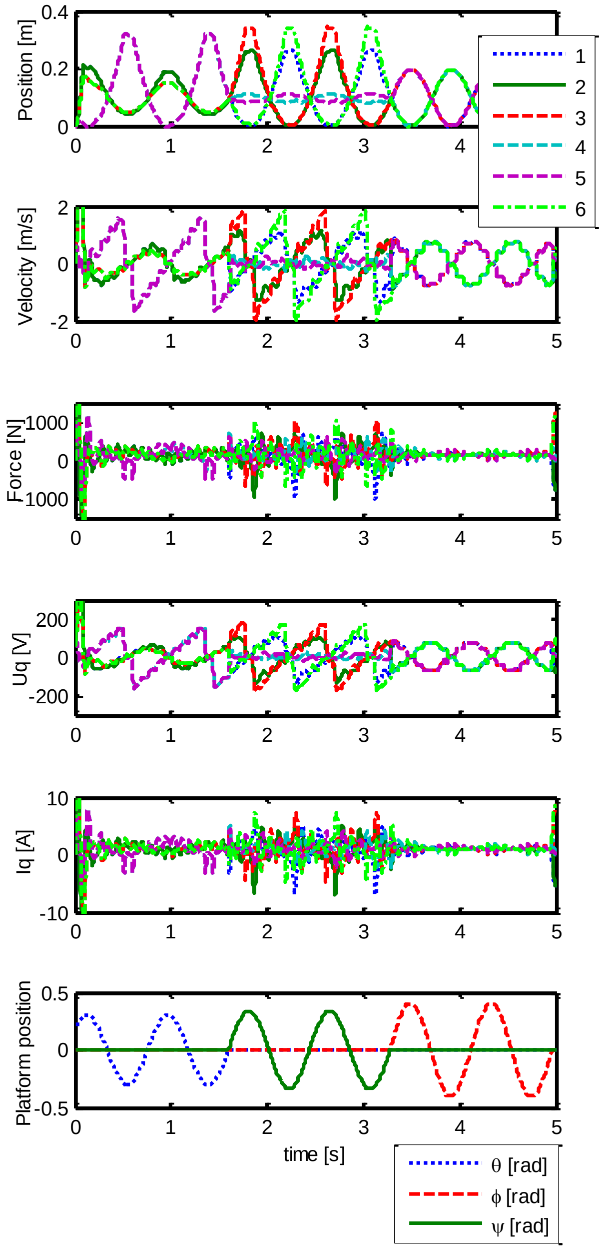

Fig. 6: Simulation Results