Computation of RMS Voltage at the Point of Common Coupling in Low Voltage Three–phase Systems for Purposes of Power Quality Evaluation

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Abstract — **This paper describes differences of two approaches for voltage computation at the point of common coupling (PCC) in three-phase low voltage (LV) systems. The first approach is based on topology of three–wire three-phase system and the second one is based on four-wire topology. The paper is aimed mainly for computation of supply voltage variations and voltage unbalance due to unsymmetrical loads. The presented method is suitable for engineers engaged in power quality evaluation.**

Keywords — Nodal analysis, power quality, voltage quality evaluation, three-phase networks modeling

I. Introduction

Many companies providing electricity distribution for end customers are increasingly interested in power quality level at the point of common coupling (PCC). One of important power quality parameters is magnitude of the supply voltage or supply voltage variations. These parameters determine minimum and maximum values of voltage during the one week measurement and are described in IEC EN 50160 standard in detail.

Validity of these parameters is often verified by measurement. But many engineers often neglect the important power network parameters in case of more detail analysis necessity, what causes failure–analysis.

It is very important to deal with the power quality even in propositions of public distribution networks. The problem of the complex power quality analysis in distribution networks using mathematical methods and models is in the variable load determination in each node.

Load variations can be determined by long time measurement or by any approximation. It is important to assume the load variations in each phase in case of approximation of the load time variability .

This paper deals with two approaches of the power quality evaluation based on one week voltage measurement (or prediction) at the supply node (supply of the transformer secondary) and the load variations modelling based on long time measurement information. The main difference between these two approaches is in assuming of phase-to-ground or phase–to–neutral voltage.

A lot of common used electric power computation software uses single–phase or three–phase three–wire approach to compute the nodal voltages or branch currents in LV electrical networks. Of course, these approaches give the same results as computations using three–phase four–wire approach in case of symmetrical voltage and impedance conditions. But if there is any unbalance in the LV four–wire system, three–phase four–wire approach should be used for correct results obtaining. The reason is that the current flows in neutral wire in case of unsymmetrical conditions. This current causes the voltage drop on the neutral wire impedance; hence the voltage measured between phase and neutral could not be the same as the voltage computed through the single–phase or three-phase three–wire modelling approach in real conditions.

The three–phase four–wire computation method described in this paper is based on nodal analysis, so nodal voltage method for three–phase approach principles are introduced in the next chapter first.

II. Nodal Voltage Method (NVM) for Three-phase Systems

The distribution of voltage and current throughout a linear power network is normally carried out using nodal analysis [1].

Application of the nodal voltage method in case of three-phase systems is basically identical to its application to single–phase networks, but it is appropriate to keep some additional principles in the topological preparation.

Also, the asymmetry inherent in transmission systems cannot be studied with any simplification using the symmetrical component frame of reference, therefore the phase components are used.

The NVM is based on the solution of equation (1):

$$\left[I_{bus}\right]=\left[Y_{bus}\right]\cdot\left[U_{bus}\right] \tag{1}$$

Consider the three–phase network shown in Fig. 1, which is a simple example of the network, where the line impedance is considered only as a parallel connection of series of *RL* components, which values are equal for Z1, Z2, Z3 and Z4.

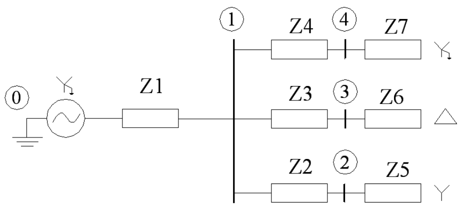

Fig. 1. Example of three-phase network – single-phase equivalent

Impedances Z5, Z6 and Z7 represent the three-phase loads with different connection, i.e. Z5 is the wye–connected load, Z6 is the delta–connected load and Z7 is the grounded wye-connected load.

One single way how to model such network is to model it using the single–phase method as describes Fig. 2. One can see the difference in number of nodes between Fig. 1 and Fig. 2. The reason is that there is one node for each phase of the network in Fig. 2 instead of one node for all three phases as shows Fig. 1. Because there are three phases for each node in Fig. 1, the real number of nodes is:

$$N_{3f} = 3 \times N_{1f} + N_y - 2 \tag{2}$$

where N_{3f} is the number of nodes of the three–phase equivalent, N_{1f} is the number of nodes of the single–phase equivalent model and N_y the is number of the wye–connected elements.

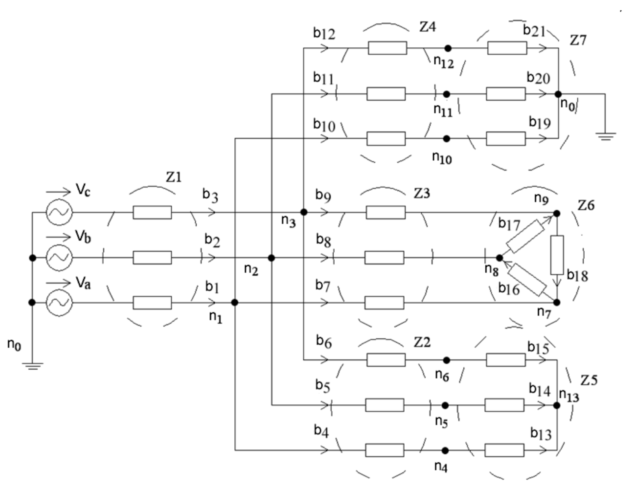

Fig. 2. Example of three-phase network – three–phase equivalent (three–wire)

The first step for the branch current and nodal voltage calculation is to correctly determine the nodal admittance matrix [Y], which can be obtained from equation (3):

$$[Y] = [A]^T [Y_d][A] \tag{3}$$

where [A] is incidence matrix, for this case:

$$[A] = \begin{array}{c|ccc|ccc|ccc|ccc|c|}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\
b_1 & 1 & 0 & 0 & & & & & & & & & & \\
b_2 & 0 & 1 & 0 & & & & & & & & & & \\
b_3 & 0 & 0 & 1 & & & & & & & & & & \\
b_4 & -1 & 0 & 0 & 1 & 0 & 0 & & & & & & & \\
b_5 & 0 & -1 & 0 & 0 & 1 & 0 & & & & & & & \\
b_6 & 0 & 0 & -1 & 0 & 0 & 1 & & & & & & & \\
b_7 & -1 & 0 & 0 & & & & 1 & 0 & 0 & & & & \\
b_8 & 0 & -1 & 0 & & & & 0 & 1 & 0 & & & & \\
b_9 & 0 & 0 & -1 & & & & 0 & 0 & 1 & & & & \\
b_{10} & -1 & 0 & 0 & & & & & & & 1 & 0 & 0 & \\
b_{11} & 0 & -1 & 0 & & & & & & & 0 & 1 & 0 & \\
b_{12} & 0 & 0 & -1 & & & & & & & 0 & 0 & 1 & \\
b_{13} & & & & -1 & 0 & 0 & & & & & & & 1 \\
b_{14} & & & & 0 & -1 & 0 & & & & & & & 1 \\
b_{15} & & & & 0 & 0 & -1 & & & & & & & 1 \\
b_{16} & & & & & & & -1 & 1 & 0 & & & & \\
b_{17} & & & & & & & 0 & -1 & 1 & & & & \\
b_{18} & & & & & & & 1 & 0 & -1 & & & & \\
b_{19} & & & & & & & & & & -1 & 0 & 0 & \\
b_{20} & & & & & & & & & & 0 & -1 & 0 & \\
b_{21} & & & & & & & & & & 0 & 0 & -1 &
\end{array} \tag{4}$$

and [Y_d] is the diagonal matrix of three–phase admittances and is determined by equation (5):

$$[Y_d] = \begin{bmatrix} Y_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Y_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Y_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & Y_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & Y_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & Y_7 \end{bmatrix} \tag{5}$$

where Y_i represents admittances of the three–phase elements and they are determined by diagonal matrices of relevant branch admittances.[2]

III. DIFFERENCE BETWEEN THREE–WIRE AND FOUR–WIRE APPROACH

Principles of the nodal voltage method for the three-phase approach were described through the three–wire power system model in the previous chapter. This approach can be used in case of the three–phase system (mainly MV networks). But in the case of LV networks four–wire systems are used mainly. Fig. 3 shows the the single–line schema of the common LV line supplying the LV customers. The customers C_i are often connected to the line through the branch lines (*BL*) in the place of the pylon *p*. The line between each pylon is called the line section (*LS*) and it is characterized by its impedance.

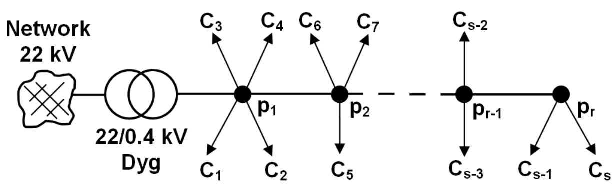

Fig. 3. Single–line schema of common LV line supplying LV customers

Each element in the single–line schema can be represented by a three–phase element in the three– or four–wire approach. Fig. 4 shows an example of topology of a small LV network represented by the external network impedance, three pylons (nodes n_2, n_3 and n_4), three line sections (LS_1, LS_2 and LS_3), five branch lines (BL_1 to BL_5) and five customers (C_1 to C_5) in the three-phase three–wire point of view. The topology of the same network in the three-phase four–wire point of view is shown in Fig. 5. The difference between these two approaches is in the existence of neutral impedance in each three–phase branch in Fig. 5. It does not matter which approach is chosen for the voltages computation in case of symmetrical conditions (symmetrical voltage source, branch impedances and loads). In case of unsymmetrical conditions (unsymmetrical voltage source or load) the the four–wire approach is much closer to real conditions.

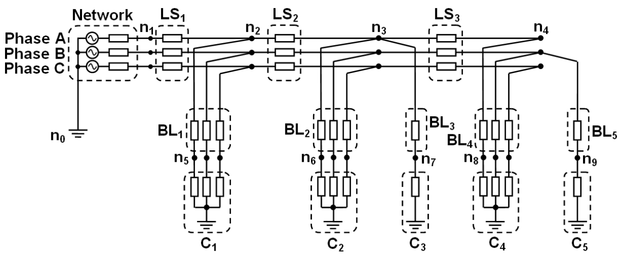

Fig. 4 Example of three-phase three-wire schema

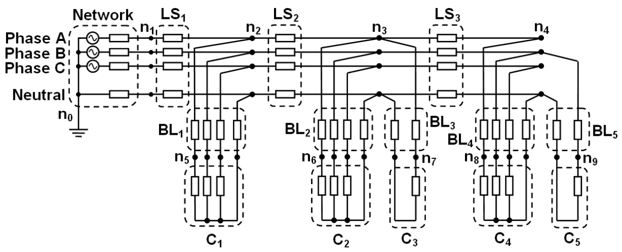

Fig. 5 Three-phase four-wire schema of network on Fig. 4

Fig. 6 shows an example of the course of a ten–minute rms voltage in each phase during one week, computed by three–wire and four-wire approach at the PCC of the last line pylon. The impedance variations of each load were determined by one-week measurement at the specific points of the LV line. One can see that the maximum and minimum of the voltage during the same week is different from the three–wire or four–wire point of view. Fig. 6 shows the voltage at one given node (PCC) in time scale. The difference between the three–wire and four–wire approach is clear also in the case of an areal scale. Fig. 7 shows voltages of each phase at the same time at each pylon of the given line. Because the voltage measurement at each point of the network is provided as the voltage–to–neutral measurement, the results of the practical measurement often lead to the similar results as shown in the four–wire approach in Fig. 7, i.e. the phase voltage in some phase increases with the length from the source (secondary of MV/LV transformer).

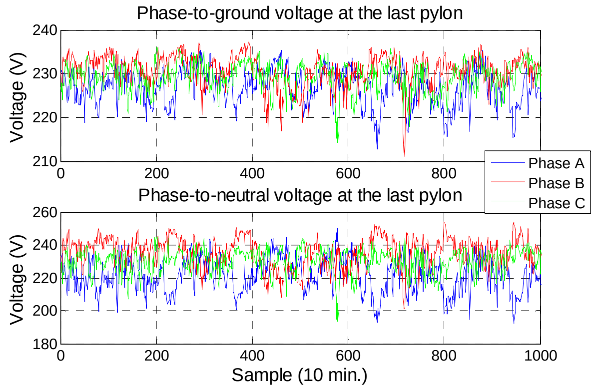

Fig. 6 One-week voltage course at the same node from the voltage–to–ground and voltage–to–neutral point of view

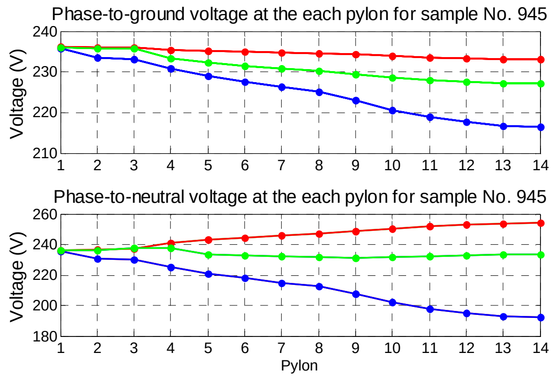

Fig. 7 Voltage propagation through the same line from the voltage-to-ground and voltage-to-neutral point of view

Because the line was modelled as the *RL*–combination instead of the pi-section (the line capacitance in case of LV line can be neglected), there was no capacitive element to increase the voltage in case of the no–load phase, and so the Ferranti effect was not the reason of voltage increasing. Many engineers accreted such increasing to error of measurement, but this is not always the reason. The effect of higher phase voltage at the end of line compared to the phase voltage at the beginning of the line (secondary of the MV/LV transformer) can be explained through next two pictures.

Fig. 8 shows a simple LV network with an ideal (symmetrical) three–phase voltage source which is connected to the three–phase symmetrical load in the first case and the single–phase load is connected in the second case. For the simplicity we assume the resistive elements only. The voltage measurement at the PCC is shown in both cases (U_{l1} – blue arrows and U_{l2} – red arrows). Source voltage in Fig. 9 is represented by three black coloured vectors. The same voltage can be measured at the PCC in the case of no load, because no current flows in the circuit – no voltage drop occurs. In the case when there is the symmetrical three–phase load connected at the PCC (first case), the sum of all three phase currents will be equal to zero, so no current will flows through the impedance of line neutral. Because no current will flow through the line neutral, there will be no voltage drop on the line neutral impedance and the voltage measured at the PCC will be symmetrical and in each phase it is smaller than the source voltage (blue arrows in Fig. 9).

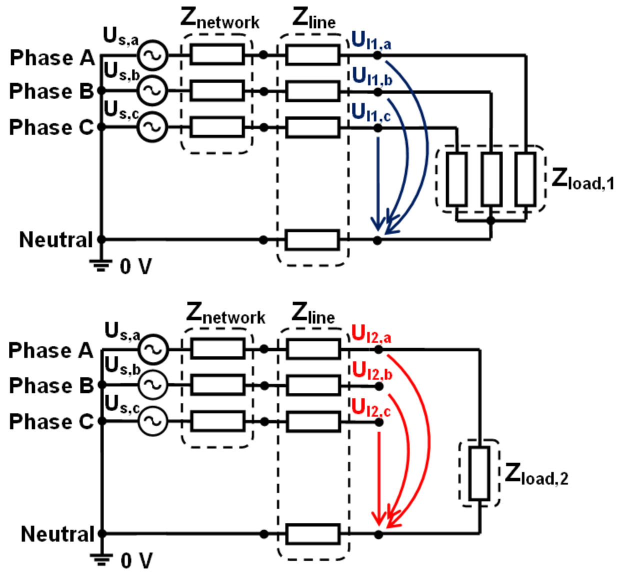

Fig. 8 Three–phase symmetrical load and single–phase load connected to the ideal three–phase voltage source

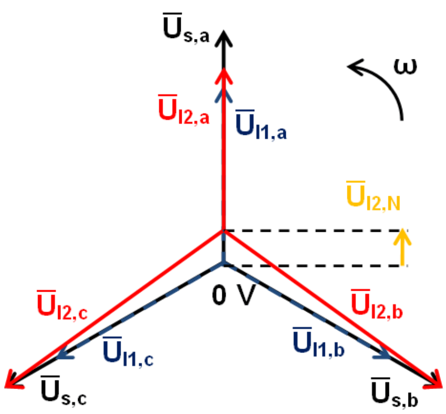

Fig. 9 Voltage phasors of Fig. 8

Different situation appears if the unsymmetrical load is connected at the PCC. This situation can be extremely represented by a single–phase load connected to the phase A at the PCC (the second case). Because no currents will flow in phases B and C, the sum of three–phase currents will not be zero. In the case of the single–phase load the same current will flows through the line neutral impedance as through the line phase impedance. In consequence of that, there will be no zero voltage in the neutral of PCC.

The direction and magnitude of the neutral–ground voltage at the PCC in such case represents the yellow arrow in Fig. 9. Because there will be no zero voltage in the neutral of PCC, voltages measured between phases and neutral in such case will not be voltages between phases and ground. Voltages measured between phases and neutral at the PCC in the case of the single–phase load are represented by the red arrows in Fig. 9. One can see that the rms voltage between the phase B and neutral, as well as the rms voltage between the phase C and neutral at the PCC is higher than the source voltage in the phase B and C.

The reason of the voltage increasing in some phase with the length from the source is the load asymmetry, which causes the current flow through the neutral and so the voltages between phases and neutral at different PCC are not voltages between phases and ground. The phasor direction of the voltage between neutral and ground depends on the phasor direction of current flowing in neutral.

CONCLUSION

Simplicity of three–phase four–wire networks to single–phase or three–phase three–wire models for many kinds of power analysis is appropriate and gives accurate enough results. But in case of the power quality analysis, these simplifications lead to incorrect conclusions due to incorrect computation approach. This paper was aimed to show the difference between the three–phase three–wire (common used) and four–wire approach in case of the power quality evaluation in LV networks. The four–wire model of LV systems should be used mainly for analyses of voltage conditions in case of unbalanced or single–phase load propositions. The evaluation of data measured in LV systems due to customer's claim to worse power quality level also cannot be studied correctly without four–wire modelling approach.

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