A Mathematical Model of Thermal Processes in the Empty Electric Muffle Furnace

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Abstract — **The paper is focused on heating within the electric resistance furnaces. Muffle resistance furnaces were chosen as a subject of the examination. The first part of the paper is aimed to describe the examined object from technology and construction points of view. The issue of a mathematical model of the muffle furnace is explained in the next chapter. The main part of the paper is aimed to divide the furnace into the lumped thermal capacitances and to create the system of temperature differential equations of state that characterize the object.**

Keywords — muffle furnace, thermal process, heating element, furnace lining and mathematical model

I. Introduction

The subject of the examination are the muffle furnaces designed for thermal processing of materials in the course of research and development, in laboratories, industrial pilot operation and also in the production of various fields at the temperature up to 1200 °C. The following examples show the possible ways how to use this kind of furnaces within various sectors:

- Health service – dental laboratories.
- Agriculture and food laboratories – combustion processes.
- Technical ceramics – raw materials calcination and samples first firing.
- Art ceramics – repeated firing and second firings of miniatures.
- Iron founding and jewellery making – melting of metals.
- Mechanical engineering – thermal refinement of smaller single parts (annealing, burning, hardening, tempering, etc.)
- Metallurgy – material samples testing, melting of some metals in special foundry crucibles.
- Chemical laboratories – drying chemicals and thermal processes.

The furnaces are not designed to be used for such processes during which flammable and explosive substances are present, possibly for the substances from which the used explosive substances may be released. The only exception is the laboratory procedure within which the samples of organic substances aimed to determine the ash content are burned. Should the gaseous products be released within the thermal process, it is necessary to place the furnace to a working place with a proper and intensive exhaust ventilation.

A furnace workspace consists of a muffle that is made of a refractory material, usually of the SiC or fireproof clay, which prevents direct contact of the furnace atmosphere with the furnace heating system. The proven furnace structured of four heated walls and distributed heating energy of the muffles ensures a homogenous distribution of the temperature field within the whole workspace. The muffle furnace is usually designed as a portable equipment.

II. Thermal Insulation

The thermal insulation of the furnace chamber is designed to follow the intended thermal process of such equipment:

1) Heating processes – this kind of heating means the furnace will be supplied with large quantity of heat and quite a high working temperature will be kept even if a cold batch is placed in. Thus the heating furnaces are equipped with a massive and insulating lining that is able to accumulate large amount of heat, which is the reason why they are of a considerable thermal inertia. This means the furnace is, from the thermal point of view, just a little sensitive to changes that occur in the course of its operation, such as a cold batch insert or a heated batch removal. Such a furnace heating up to the working temperature, or furnace cooling, when it is being shut down, takes a long time and during such period the massive lining accepts or radiates the accumulated heat.
2) Heat treatment processes – they are required to have a minimum thermal inertia so as the furnace would be able to copy the programme temperature-time curve, i.e. with the required heat-up and cooling rates. Thus the furnace will be equipped with a light, usually fibrous lining with the lowest possible heat accumulation. The furnace heating system designed for the heat treatment will have a wide range of adjustability to be able to reach the desired temperature course. Heat treatment working temperatures are lower than in the case of reheating furnaces, which results in not so complicated engineering solution of the furnace design.

The mathematical model, which was developed, results from a muffle furnace function sample that was manufactured for such purposes, i.e. to prove the results found out and based on the solution of such a model.

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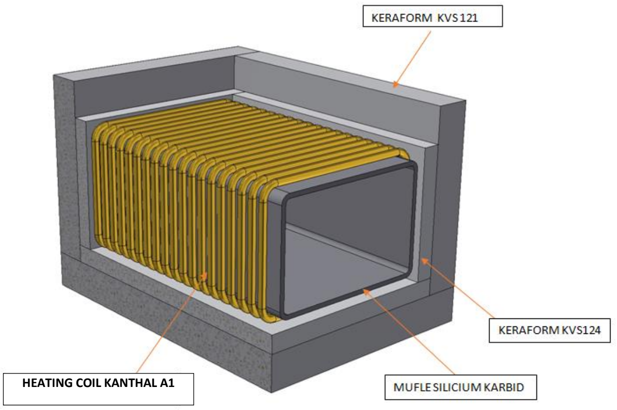

Fig. 1. A cut-view of muffle furnace sample.

III. Main Parts of the Furnace Sample

Main parts of the furnace sample are as follows:

a) Lining, or side walls, ceiling, furnace hearth, back wall and front door.
b) Ceramic muffle.
c) Heating element (heating coil).
d) Temperature sensor.

To a): The lining consists of thermal insulation which is used to maximize the thermal resistance to the heat flowing from the furnace into its ambient area. The insulation is usually made of thermal insulation (usually ceramic one) materials. The present case means we work with two material variants:

- Fibre ceramic boards of Sibral type.
- Dense fireproof clay – S1.

To b): The furnace workspace is limited by the ceramic muffle into which the material to be processed is placed – a batch. The muffle is a thin-walled case made of a suitable ceramic material. In our case two variants of material are again considered:

- Silicon carbide (SiC).
- Dense fireproof clay S1.

To c): The heating element is designed as the heating coil made of Kanthal resistance material. The heating coil is placed on the side walls, on the ceiling and furnace hearth between the muffle and thermal insulation lining. This means that the heat which is supplied by the heating coil is directed both into the muffle and from here goes to the furnace workspace and also into the lining and from here to the ambient area of the furnace. The heat permeates through the back wall and the door from the furnace workspace into its ambient area as the waste heat.

To d): The temperature in the furnace is usually ensured using the thermo-electric temperature sensors that are inserted using the furnace wall into the workspace. These sensors are designed as the rod thermo-electric thermometers with thermocouples placed in the coaxial metal and ceramic protection tubes or as the sheathed thermocouples of a significantly smaller diameter than in the case of rod thermometers. One of the characteristics of the sheathed thermocouples is their fast response to a temperature change.

The temperature detected by this kind of sensors shows in the operating practice the temperature of the furnace workspace. It is necessary to realize that such a value corresponds only to the temperature of the measuring joint (welded joint of a thermocouple) and such a temperature value results from the heat interchange from the non-homogeneous temperature field that surrounds the sensor in the furnace.

IV. The Principle of Electric Furnace Mathematical Model

A furnace – it is the heating system with continuously distributed parameters. The mathematical description of the performance of the furnace temperatures and heat flows may be carried out in time using a system of partial differential equations of the heat interchange.

For our purposes we can simplify and replace the continuous continuum with a limited number of lumped parameters. Such a system can be described as a system of usual differential equations of the 1st order, where the arbitrary variable X means the time, and dependent variable Y means the temperature.

From the practice point of view we can proceed as follows: the furnace walls will be divided into an adequate number of layers. The characteristics of the particular layers will be concentrated into one thermal capacitance that is surrounded by a few thermal resistances. Each thermal capacitance status is described by its temperature. The heat flows are flowing between the individual capacitances and through the thermal resistances.

The thermal capacitances value of particular layers is determined from its mass and its specific thermal capacitance.

The value of the thermal resistance is determined:

- By the heat interchange in a solid body: by the thickness of material layer, its heat conductivity and heat interchange area.
- By the heat interchange between the surface of a solid body and its ambient environment: by the heat interchange area, with a coefficient of convective heat passage and relations of the heat interchange through the heat emission.

The values of the heat flows between the capacitances are based on the temperature differences between the capacitances and thermal resistances that are between them.

The input value which can affect or control the whole system performance, is the electric input of the heating coil, which is expressed as the heat flow (W).

Some of the physical values that are present in the model may be in the whole temperature working range considered to be constants (masses, interchange areas, thicknesses of layers). The values of other physical parameters may vary within the temperature working range, which happens usually depending on the temperatures (heat conductivity coefficients, specific heat,

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coefficients of heat passage, furnace air mass). Such variability will be respected by the model.

From the manageability point of view related to the mathematical model it is appropriate to perform the division so that the number of the lumped capacitances would not overcome the value of a few tens.

V. The Method of Distribution of a Furnace into Lumped Thermal Capacitances

The heating coil is considered to be one of the thermal capacitance which has its own mass and specific thermal capacitance.

The thermal insulation lining of the sides, ceiling and furnace hearth will be considered as symmetric and their characteristics will be put together and split into four layers from which each layer will represent one lumped capacitance of the mass that is equal to the layer mass. The temperature of the lumped capacitance then characterizes the thermal status of the layer and corresponds approximately to the temperature that is in the middle of the layer.

The back wall and the door will be deemed as mutually symmetric, their characteristics are put together and then divided into five layers, from which each layer will represent one lumped capacitance.

The muffle is then split into two layers, from which each layer will represent one lumped capacitance.

The gaseous content of the furnace will be deemed as one thermal capacitance, where its mass and also the specific thermal capacitance are being changed together with the temperature.

The measuring end of a sheathed thermocouple, i.e. a temperature sensor, will be also considered to be one thermal capacitance.

The heat passage from the solid bodies into the internal and external surroundings depends on the surface temperatures of the bodies. For such a reason we choose to divide the thickness of the lining and muffle layers so as the layers would be in the wall depth thicker and the layer on the surface would be so thin that its temperature may be considered as the surface one.

Within such a division of the furnaces we get 14 lumped thermal capacitances.

VI. Heat Interchange between the Thermal Capacitances

As a result of different temperatures the heat interchange between individual thermal capacitances is being performed.

Inside the solid bodies (internal heat interchange) – by the conduction between the surfaces of the bodies and their ambient areas – by the convection and radiation (external interchange).

The internal heat interchange, i.e. the heat flows between the adjacent thermal capacitances of the object layers, is solved as a steady one-dimensional heat interchange through the flat wall that consists of half-thickness of both adjacent layers.

The heat flow of the external interchange is the sum of the convection heat flow between the surface and its gaseous surroundings and also of the resultant radiation heat flow between the surface and other surfaces that are surrounding it. The gaseous environment inside the furnace and also in its ambient area is considered to be diathermal, as these are mostly diatomic gases. Thus, it does not affect the radiation heat transfer.

A thin sheathed thermocouple may be considered to be a thermally thin body that has in the entire volume the same temperature. The heat is spread into the thermocouple through the convection from gas in the furnace and through radiation from surface areas – inside muffle surface and inside door surface and furnace back wall.

VII. Representation of Mathematical Model Structure

For a graphical representation of the furnace mathematical model structure may be used the tools that are usual in the field of electrical engineering when presenting electric circuits.

Analogy:

Electrical capacitance	–	Thermal capacitance
Electrical resistance	–	Thermal resistance
Electric current	–	Heat flow
Voltage	–	Temperature

Using this kind of analogy we get the following diagram of the above described muffle furnace mathematical model (see Fig. 2).

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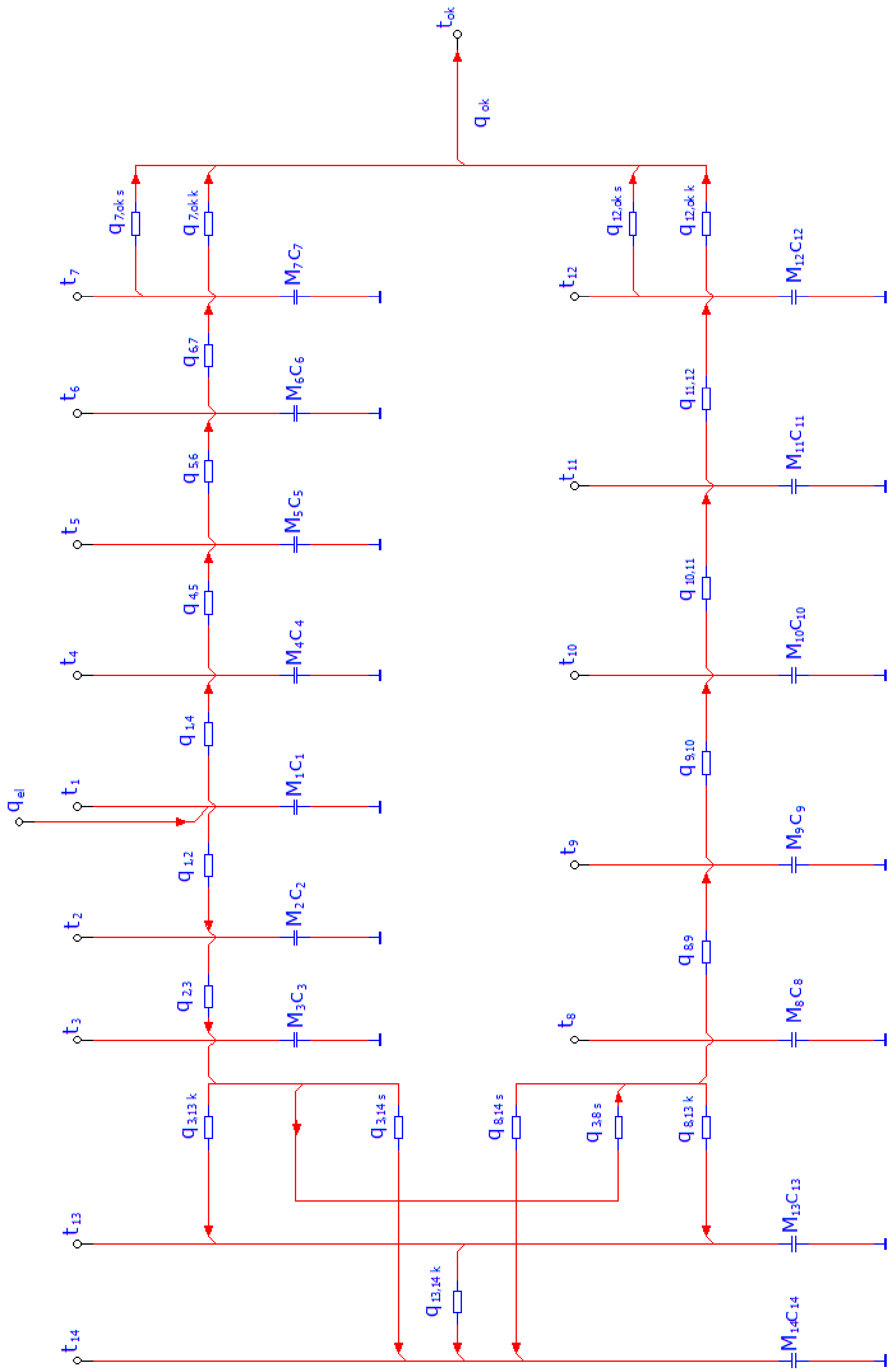

Fig. 2. Structure of furnace mathematical model.

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TABLE I.
DESIGNATION OF QUANTITIES WITHIN THE FURNACE MATHEMATICAL MODEL

Quantity name	Designation	Dimension symbol
Time	T	(s)
Thermal capacitances mass	M_i	(kg)
Furnace gas mass	M_{13}	(kg)
Thickness of the layers	s_i	(m)
Thermal conductivities	λ_i	(W/mK)
Specific thermal capacitances	c_i	(J/kgK)
Temperatures of individual layer capacitance	t_i	°C
Heat flow from the i-capacitance into the j-capacitance	$q_{i,j}$	(W)
Heating coil electrical output	q_{el}	(W)
Heat flow from the i-capacitance into the ambient area	$q_{i,\mathrm{ok}}$	(W)
Surfaces of heat passage from the i-capacitance into the j-capacitance	$S_{i,j}$	(m^2)
Coefficient of heat passage through convection on the i-surface	$\alpha_{\mathrm{k}i}$	(W/m^2K)
Variable emissivity between the i-surface and the j-surface	$\varepsilon_{i,j}$	(1)
Angular coefficient of the exposure to radiation from the i-surface to the j-surface	$\varphi_{i,j}$	(1)

TABLE II. – PART 1
QUANTITIES INDEXING WITHIN THE FURNACE MATHEMATICAL MODEL

Index	Referred to:
1	Heating coil
	Muffle:
2	Muffle layer next to the heating coil
3	Muffle layer next to the inside furnace surface
	Side walls + ceiling + furnace hearth:
4	The first layer of insulation – following the heating coil
5	The second layer of insulation – the out direction
6	The third layer of insulation - the out direction
7	The fourth layer of insulation – surface one
	Door + back wall:
8	The first layer of insulation – following the inside furnace surface
9	The second layer of insulation – the out direction
10	The third layer of insulation – the out direction
11	The fourth layer of insulation – the out direction
12	The fifth layer of insulation – surface one
13	Gas in the furnace
14	The end of a sheathed thermocouple (convection)
15	The end of a sheathed thermocouple (radiation)
16	Furnace ambient area (convection)
17	Furnace ambient area (radiation)

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TABLE II. – PART 2
QUANTITIES INDEXING WITHIN THE FURNACE MATHEMATICAL MODEL

Index	Referred to:
Ok	Furnace ambient area
El	Electric power source
S	Radiant
K	Convection

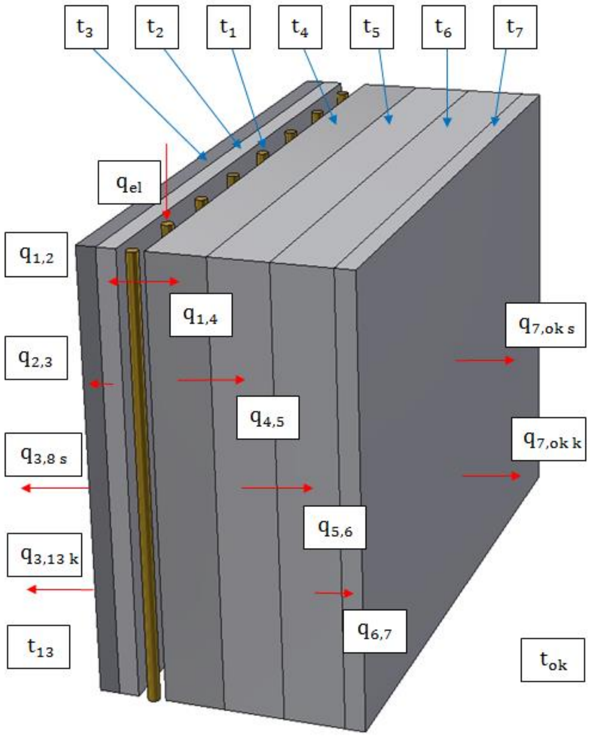

Fig. 3. Demonstration of heat flows in the side walls, ceiling and furnace hearth.

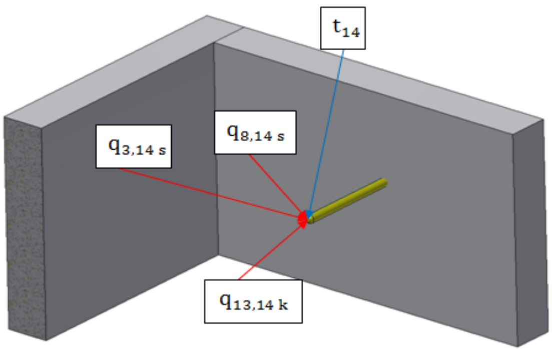

Fig. 5. Demonstration of heat flows affecting the temperature sensor.

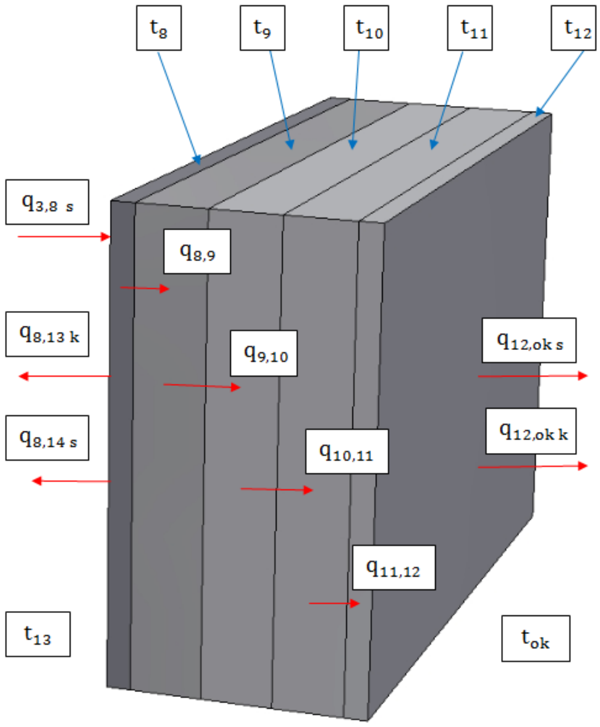

Fig. 4. Demonstration of heat flows and temperatures within the model back wall and door.

VIII. INPUTS AND OUTPUTS OF THE MATHEMATICAL MODEL

The input, i.e. independent value, is the electric input into the heating coil $q_{el} = q_{el}(\tau)$.

The model output may be any of the variable values that are present within this model.

The interesting output values include:

t_{13} – Gas temperature in the furnace.
t_{14} – Temperature of thermoelectric sensor.
t_7, t_{12} – Temperatures of outside surface of the furnace.
q_{ok} – Heat flow into the furnace ambient area.

For the steady state temperature it will be $q_{el} = q_{ok}$.

IX. A SYSTEM OF TEMPERATURE DIFFERENTIAL EQUATIONS OF STATE

Generally, it will be valid, that the change of thermal content of the capacitance is given by the sum of the input and output heat.

Heating coil:

$$M_1 c_1 \frac{\mathrm{d}t_1}{\mathrm{d}\tau} = q_{el} - q_{1,2} - q_{1,4} \ . \tag{1}$$

Muffle:

$$M_2 c_2 \frac{\mathrm{d}t_2}{\mathrm{d}\tau} = q_{1,2} - q_{2,3} \ , \tag{2}$$

$$M_3 c_3 \frac{\mathrm{d}t_3}{\mathrm{d}\tau} = q_{2,3} - q_{3,8\,\mathrm{s}} - q_{3,13\,\mathrm{k}} - q_{3,14\,\mathrm{s}} \ . \tag{3}$$

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Lining of side walls + ceiling + furnace hearth:

$$M_4 c_4 \frac{dt_4}{d\tau} = q_{1,4} - q_{4,5} \ , \quad (4)$$

$$M_5 c_5 \frac{dt_5}{d\tau} = q_{4,5} - q_{5,6} \ , \quad (5)$$

$$M_6 c_6 \frac{dt_6}{d\tau} = q_{5,6} - q_{6,7} \ , \quad (6)$$

$$M_7 c_7 \frac{dt_7}{d\tau} = q_{6,7} - q_{7,\mathrm{ok\,k}} - q_{7,\mathrm{ok\,s}} \ . \quad (7)$$

Lining of door + back wall:

$$M_8 c_8 \frac{dt_8}{d\tau} = q_{3,8\,\mathrm{s}} - q_{8,9} - q_{8,13\,\mathrm{k}} - q_{8,14\,\mathrm{s}} \ , \quad (8)$$

$$M_9 c_9 \frac{dt_9}{d\tau} = q_{8,9} - q_{9,10} \ , \quad (9)$$

$$M_{10} c_{10} \frac{dt_{10}}{d\tau} = q_{9,10} - q_{10,11} \ , \quad (10)$$

$$M_{11} c_{11} \frac{dt_{11}}{d\tau} = q_{10,11} - q_{11,12} \ , \quad (11)$$

$$M_{12} c_{12} \frac{dt_{12}}{d\tau} = q_{11,12} - q_{12,\mathrm{ok\,k}} - q_{12,\mathrm{ok\,s}} \ . \quad (12)$$

Gas in the furnace:

$$M_{13} c_{13} \frac{dt_{13}}{d\tau} = q_{3,13\,\mathrm{k}} + q_{8,13\,\mathrm{k}} - q_{13,14\,\mathrm{k}} \ . \quad (13)$$

Thermometer:

$$M_{14} c_{14} \frac{dt_{14}}{d\tau} = q_{13,14\,\mathrm{k}} + q_{3,14\,\mathrm{s}} + q_{8,14\,\mathrm{s}} \ . \quad (14)$$

A total of 14 ordinary differential equations of the 1st order.

A total of 22 relations for heat flows determination.

X. Relations for Heat Flows

$$q_{\mathrm{el}} = q_{\mathrm{el}}(\tau) \quad (15)$$

$$q_{1,2} = S_{1,2}(t_1 - t_2)/(s_2/2\lambda_2) \quad (16)$$

$$q_{2,3} = S_{2,3}(t_2 - t_3)/(s_2/2\lambda_2 + s_3/2\lambda_3) \quad (17)$$

$$q_{3,13\,\mathrm{k}} = S_{3,13}(t_3 - t_{13})\alpha_{\mathrm{k3}} \quad (18)$$

$$q_{3,8\,\mathrm{s}} = \varepsilon_{\check{c}}\,\varepsilon_{3,8}\left((t_3+273)^4 - (t_8+273)^4\right)S_{3,8}\varphi_{3,8} \quad (19)$$

$$q_{3,14\,\mathrm{s}} = \varepsilon_{\check{c}}\,\varepsilon_{3,14}\left((t_3+273)^4 - (t_{14}+273)^4\right)S_{3,14}\varphi_{3,14} \quad (20)$$

$$q_{1,4} = S_{1,4}(t_1 - t_4)/(s_4/2\lambda_4) \quad (21)$$

$$q_{4,5} = S_{4,5}(t_4 - t_5)/(s_4/2\lambda_4 + s_5/2\lambda_5) \quad (22)$$

$$q_{5,6} = S_{5,6}(t_5 - t_6)/(s_5/2\lambda_5 + s_6/2\lambda_6) \quad (23)$$

$$q_{6,7} = S_{6,7}(t_6 - t_7)/(s_6/2\lambda_6 + s_7/2\lambda_7) \quad (24)$$

$$q_{7,\mathrm{ok\,k}} = S_{7,\mathrm{ok}}(t_7 - t_{\mathrm{ok}})\alpha_{\mathrm{k7}} \quad (25)$$

$$q_{7,\mathrm{ok\,s}} = \varepsilon_{\check{c}}\,\varepsilon_{7,\mathrm{ok}}\left((t_7+273)^4 - (t_{\mathrm{ok}}+273)^4\right)S_{7,\mathrm{ok}}\varphi_{7,\mathrm{ok}} \quad (26)$$

$$q_{8,9} = S_{8,9}(t_8 - t_9)/(s_8/2\lambda_8 + s_9/2\lambda_9) \quad (27)$$

$$q_{8,13\,\mathrm{k}} = S_{8,13}(t_8 - t_{13})\alpha_{\mathrm{k8}} \quad (28)$$

$$q_{8,14\,\mathrm{s}} = \varepsilon_{\check{c}}\,\varepsilon_{8,14}\left((t_8+273)^4 - (t_{14}+273)^4\right)S_{8,14}\varphi_{8,14} \quad (29)$$

$$q_{9,10} = S_{9,10}(t_9 - t_{10})/(s_9/2\lambda_9 + s_{10}/2\lambda_{10}) \quad (30)$$

$$q_{10,11} = S_{10,11}(t_{10} - t_{11})/(s_{10}/2\lambda_{10} + s_{11}/2\lambda_{11}) \quad (31)$$

$$q_{11,12} = S_{11,12}(t_{11} - t_{12})/(s_{11}/2\lambda_{11} + s_{12}/2\lambda_{12}) \quad (32)$$

$$q_{12,\mathrm{ok\,k}} = S_{12,\mathrm{ok}}(t_{12} - t_{\mathrm{ok}})\alpha_{\mathrm{k12}} \quad (33)$$

$$q_{12,\mathrm{ok\,s}} = \varepsilon_{\check{c}}\,\varepsilon_{12,\mathrm{ok}}\left((t_{12}+273)^4 - (t_{\mathrm{ok}}+273)^4\right)S_{12,\mathrm{ok}}\varphi_{12,\mathrm{ok}} \quad (34)$$

$$q_{13,14\,\mathrm{k}} = S_{13,14}(t_{13} - t_{14})\alpha_{\mathrm{k14}} \quad (35)$$

$$q_{\mathrm{ok}} = q_{7,\mathrm{ok\,k}} + q_{7,\mathrm{ok\,s}} + q_{12,\mathrm{ok\,k}} + q_{12,\mathrm{ok\,s}} \quad (36)$$

XI. Conclusion

The above described mathematical model is suitable for numerical simulation calculations of thermal processes that are performed during the heating and cooling of the electric muffle furnaces. It allows to give detailed information about the time course of the temperatures and, especially, about the heat flows, which are very difficult to measure in a real furnace, if not impossible. The results of the simulations with the above described model have proven very good conformity with the measured results that were performed in real furnaces. I describe a comparison in the article “Experimental Verification of Model Simulation Results of Heating Cycles within the Electric Muffle Furnaces”. The mathematical model of the muffle furnace can, in this case, become an effective tool for an efficient design of such equipment. From the technology needs point of view this will allow to adjust the design to the requirements, in particular to minimize high energy consumption of its operation.

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