

M2-Branes in $\mathcal{N} = 3$ Harmonic Superspace

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Abstract

We give a brief account of the recently proposed $\mathcal{N} = 3$ superfield formulation of the $\mathcal{N} = 6, 3D$ superconformal theory of Aharony *et al* (ABJM) describing a low-energy limit of the system of multiple M2-branes on the $AdS_4 \times S^7/\mathbb{Z}_k$ background. This formulation is given in harmonic $\mathcal{N} = 3$ superspace and reveals a number of surprising new features. In particular, the sextic scalar potential of ABJM arises at the on-shell component level as the result of eliminating appropriate auxiliary fields, while there is no explicit superpotential at the off-shell superfield level.

1 Preliminaries: AdS/CFT

1.1 AdS/CFT in IIB superstring

As the starting point, I recall the essentials of the original AdS/CFT correspondence (for details see [1] and references therein).

It is the conjecture that the IIB superstring on $AdS_5 \times S^5$ is in some sense dual to maximally supersymmetric $\mathcal{N} = 4, 4D$ super Yang-Mills (SYM) theory. This hypothesis is to a large extent based upon the coincidence of the symmetry groups of both theories. Indeed,

$$AdS_5 \times S^5 \sim \frac{SO(2,4)}{SO(1,4)} \times \frac{SO(6)}{SO(5)} \subset \frac{SU(2,2|4)}{SO(1,4) \times SO(5)},$$

so the superisometries of this background constitute the supergroup $SU(2,2|4)$. On the other hand, the supergroup $SU(2,2|4)$ defines superconformal invariance of $\mathcal{N} = 4$ SYM, with $SO(2,4)$ and $SO(6) \sim SU(4)$ being, respectively, $4D$ conformal group and R-symmetry group.

Some related salient features of the AdS/CFT correspondence are as follows.

- $AdS_5 \times S^5$ (plus a constant closed 5-form on S^5) is the bosonic “body” of the maximally supersymmetric curved solution $\frac{SU(2,2|4)}{SO(1,4) \times SO(5)}$ of IIB, $10D$ supergravity. It preserves **32** supersymmetries.
- $\mathcal{N} = 4$ SYM action with the gauge group $U(N)$ is the low-energy limit of a gauge-fixed action of a stack of N coincident D3-branes on $AdS_5 \times S^5$: **4** worldvolume co-ordinates of the latter system become the Minkowski space-time co-ordinates, while **6** transverse ($u(N)$ algebra-valued) D3-brane co-ordinates yield just **6** scalar fields of the nonabelian $\mathcal{N} = 4, 4D$ gauge multiplet.
- This system has the following *on-shell* content: **6** bosons and **16/2 = 8** fermions (all $u(N)$ algebra

valued); **2** “missing” bosonic degrees of freedom which are required by world-volume $\mathcal{N} = 4$ supersymmetry come from a gauge field. This is a “heuristic” explanation why just D3-branes, with the gauge fields contributing non-trivial degrees of freedom on shell, matter in the case of the AdS_5/CFT_4 correspondence.

1.2 AdS/CFT in M-theory

Recently, there has been a surge of interest in another example of AdS/CFT duality, this time related to M-theory and the IIA superstring.

The fundamental (though not explicitly formulated as yet) M-theory can be defined as a strong-coupling limit of the IIA, $10D$ superstring with $11D$ supergravity as the low-energy limit. It has the following maximally supersymmetric classical curved solution:

$$AdS_4 \times S^7 \sim \frac{SO(2,3)}{SO(1,3)} \times \frac{SO(8)}{SO(7)} \subset \frac{OSp(8|4)}{SO(1,3) \times SO(7)}$$

(plus a constant closed 7-form on S^7), which preserves **32** supersymmetries.

When trying to treat this option within the general AdS/CFT correspondence (like the previously discussed $AdS_5 \times S^5$ example), there arise the following natural questions.

- What is the CFT dual to this geometry?
 1. It should be some $3D$ analog of $\mathcal{N} = 4$ SYM and should arise as a low-energy limit of multiple M2-branes (membranes of M-theory, analogs of D3-branes of the IIB superstring).
 2. Hence it should contain **8** (gauge algebra valued) scalar fields which originate from the transverse co-ordinates of M2-branes.
 3. It should contain off-shell **16** physical fermions (**16** other fermionic modes can be gauged away by the relevant κ symmetry).

4. Finally, it should be superconformal, with $OSp(8|4)$ realized as $\mathcal{N} = 8, 3D$ superconformal group.

- On shell there should be $8 + 16/2 = 8 + 8$ degrees of freedom. Hence the gauge fields should not contribute any degree of freedom on shell in this special case (in a drastic contrast with the “type IIB / $\mathcal{N} = 4$ SYM” correspondence).

The unique possibility which meets all these demands is that the dual theory is some supersymmetric extension of *Chern-Simons* gauge theory [2].

2 Chern-Simons theories

The standard bosonic Chern-Simons (CS) action is as follows

$$S_{cs} = \frac{k}{4\pi} \text{Tr} \int d^3x \epsilon^{mns} \left(A_m \partial_n A_s + \frac{2i}{3} A_m A_n A_s \right) \quad (2.1)$$

$$\Rightarrow \mathcal{F}_{mn} = \partial_m A_n - \partial_n A_m + [A_m, A_n] = 0,$$

i.e. the YM field A_n is pure gauge on shell.

The $\mathcal{N} = 1$ superextension of the CS action is obtained by extending A_n to $\mathcal{N} = 1$ gauge supermultiplet

$$A_n \Rightarrow (A_n, \chi^\alpha), \quad \alpha = 1, 2; \quad (2.2)$$

$$\mathcal{L}_{cs}(A) \Rightarrow \mathcal{L}_{cs}(A) - \text{Tr}(\bar{\chi}\chi).$$

The fermionic field χ is auxiliary, and no dynamical (Dirac) equation for it appears. The same phenomenon takes place in the case of $\mathcal{N} = 2$ and $\mathcal{N} = 3$ superextensions of the pure CS action. The physical fermionic fields (having standard kinetic terms) can appear only from the matter supermultiplets coupled to the CS one.

Keeping in mind these general properties of supersymmetric Chern-Simons theories, Schwarz assumed [2] that the theory dual to $\text{AdS}_4 \times S^7$ must be $\mathcal{N} = 8$ superextension of the $3D$ CS theory, i.e. one should deal with the on-shell supermultiplet $(A_m, \phi^I, \psi_\alpha^B)$, $I = 1, \dots, 8$, $B = 1, \dots, 8$.

How to gain physical kinetic terms for **16** ($u(N)$ algebra-valued) fermions? The recipe: place the latter into matter multiplets of the manifest $\mathcal{N} = 1$, $\mathcal{N} = 2$ or $\mathcal{N} = 3$ supersymmetries, consider the relevant combined “CS + matter” actions and realize extra supersymmetries as the hidden ones mixing the CS supermultiplet with the matter multiplets.

3 BLG and ABJM models

3.1 Attempts toward $\mathcal{N}=8$ CS theory

The first attempt to formulate the appropriate CS theory was undertaken by J. Schwarz in 2004 [2]. He used $\mathcal{N} = 2, 3D$ superfield formalism and tried to construct

$\mathcal{N} = 8$ superconformal CS theory as $\mathcal{N} = 2$ CS theory plus **4** complex matter chiral superfields (with the off-shell content consisting of **8** physical bosons, **16** fermions and **8** auxiliary fields). However, these attempts failed. As became clear later, the reason for this failure is that the standard assumption that both matter and gauge fields are in the adjoint of the gauge group prove to be wrong in this specific case.

Such a theory was constructed by Bagger and Lambert [3] and Gustavsson [4]. The basic assumption of BLG was that the scalar fields and fermions take values in an unusual “three-algebra”

$$[T_a, T_b, T_c] = f_{abc}{}^d T_d. \quad (3.1)$$

The gauge group acts as automorphisms of this algebra, gauge fields being still in the adjoint. The totally antisymmetric “structure” constants of the 3-algebra should satisfy a fundamental Jacobi-type identity

$$f_{abc}{}^d f^{egh}{}_d + \text{some permutations of indices} = 0. \quad (3.2)$$

BLG managed to define $\mathcal{N} = 8$ (on-shell) supersymmetry in such a system and to construct the invariant Lagrangian

$$\mathcal{L}_{\mathcal{N}=8} = \tilde{\mathcal{L}}_{cs}(A) + \text{covariantized kin. terms of } \phi^I, \psi^A + \text{6-th order potential of } \phi^I + \dots,$$

where $\tilde{\mathcal{L}}_{cs}(A)$ is some generalization of the Lagrangian in (2.1). All terms involve the constants $f_{abc}{}^d$ and contain only one free parameter, the CS level k .

3.2 Problems with the BLG construction

Assuming that the 3-algebra is finite-dimensional and no ghosts are present among the scalar fields, the only solution of the fundamental identity (3.2) proved to be $f^{abcd} = \epsilon^{abcd}$, $a, b = 1, 2, 3, 4$.

Thus the only admissible gauge group is $SO(4) \sim SU(2)_L \times SU(2)_R$ and ϕ^I, ψ^A are in the “bi-fundamental” representation of this gauge group (in fact these are just $SO(4)$ vectors). No generalization to the higher-dimensional gauge groups with the finite number of generators and positive-defined Killing metric is possible.

The $SU(2) \times SU(2)$ gauge group case can be shown to correspond just to two M2-branes. How to describe the system of N M2-branes?

3.3 Way out: ABJM construction

Aharony, Bergman, Jafferis, Maldacena in 2008 [5] proposed a way to evade this restriction on the gauge group. Their main observation was that there is no need in exotic 3-algebras to achieve this at all! The fields ϕ^I, ψ^A should be always in the bi-fundamental

of the gauge group $U(N) \times U(N)$, while the double set of gauge fields should be in the adjoint.

The ABJM theory is in fact dual to M-theory on $\text{AdS}_4 \times S^7/\mathbb{Z}_k$, and in general it respects only $\mathcal{N} = 6$ supersymmetry and $SO(6)$ R-symmetry. The invariant action is a low-energy limit of the worldvolume action of N coincident M2-branes on this manifold.

For the gauge group $SU(2) \times SU(2)$, the ABJM theory is equivalent to the BLG theory.

The full on-shell symmetry of the ABJM action is the $\mathcal{N} = 6, 3D$ superconformal symmetry $OSp(6|4)$. Characteristic features of this action are the presence of sextic scalar potential of special form and the absence of any free parameter except for the CS level k . This k is common for both $U(N)$ CS actions which should appear with the relative sign minus (only in this case there is an invariance under $\mathcal{N} = 6$ supersymmetry).

3.4 Superfield formulations

Off-shell superfield formulations make manifest underlying supersymmetries and frequently reveal unusual geometric properties of supersymmetric theories. Thus it was advantageous to find a superfield formulation of the ABJM model with the maximal number of supersymmetries being manifest and off-shell.

$\mathcal{N} = 1$ and $\mathcal{N} = 2$ off-shell superfield formulations were given in refs. [6, 7, 8]. They allowed one to partly clarify the origin of the interaction of scalar and spinor component fields. On-shell $\mathcal{N} = 6$ and $\mathcal{N} = 8$ formulations were also constructed for both the ABJM and BLG models (see e.g. [9, 10, 11]).

The maximally possible off-shell supersymmetry for the CS theory coupled to matter is $\mathcal{N} = 3, 3D$ supersymmetry [12, 13]. Thus it was an urgent problem to reformulate the general ABJM models in $\mathcal{N} = 3, 3D$ superspace. This was recently done in [14].

This formulation uses the $\mathcal{N} = 3, 3D$ version [12] of the $\mathcal{N} = 2, 4D$ harmonic superspace [15, 16].

4 $\mathcal{N} = 3$ superfield formulation of the ABJM model

4.1 $\mathcal{N} = 3, 3D$ harmonic superspace

$\mathcal{N} = 3, 3D$ harmonic superspace (HSS) is an extension of the standard real $\mathcal{N} = 3, 3D$ superspace by the harmonic variables parametrizing the sphere $S^2 \sim SU(2)_R/U(1)_R$:

$$\begin{aligned} (x^m, \theta_\alpha^{(ik)}) &\Rightarrow (x^m, \theta_\alpha^{(ik)}, u_j^\pm), \\ u_i^\pm &\in SU(2)_R/U(1)_R, \quad u^{+i}u_i^- = 1, \\ m, n &= 0, 1, 2; \quad i, k, j = 1, 2; \quad \alpha = 1, 2. \end{aligned} \quad (4.1)$$

The most important feature of the $\mathcal{N} = 3, 3D$ HSS is the presence of an analytic subspace in it, with a lesser

number of Grassmann variables (two $3D$ spinors as opposed to three such spinor coordinates of the standard superspace)

$$\begin{aligned} (\zeta^M) &\equiv (x_A^m, \theta_\alpha^{++}, \theta_\alpha^0, u_k^\pm), \\ \theta_\alpha^{++} &= \theta_\alpha^{(ik)}u_i^+u_k^+, \quad \theta_\alpha^0 = \theta_\alpha^{(ik)}u_i^+u_k^-. \end{aligned} \quad (4.2)$$

It is closed under both the $\mathcal{N} = 3, 3D$ Poincaré supersymmetry and its superconformal extension $OSp(3|4)$.

All the basic objects of the $\mathcal{N} = 3$ superspace formulation live as unconstrained superfields on this subspace:

1. Gauge superfields

$$\begin{aligned} V^{++}(\zeta), \quad \delta V^{++} &= -\mathcal{D}^{++}\Lambda(\zeta) - [V^{++}, \Lambda], \\ \Lambda &= \Lambda(\zeta). \end{aligned} \quad (4.3)$$

2. Matter superfields (hypermultiplets)

$$\begin{aligned} (q^+(\zeta), \bar{q}^+(\zeta)), \\ q^+ = u_i^+ f^i + (\theta^{++\alpha} u_k^- - \theta^{0\alpha} u_k^+) \psi_\alpha^k + \infty \\ \text{of aux. fields.} \end{aligned} \quad (4.4)$$

In eq. (4.3), \mathcal{D}^{++} is the analyticity-preserving derivative on the harmonic sphere S^2 .

4.2 $\mathcal{N} = 3$ action

The $\mathcal{N} = 3$ superspace formulation of the $U(N) \times U(N)$ ABJM model [14] involves:

1. The gauge superfields V

As a result, the equations of motion are written solely in terms of analytic superfields in the simple form:

$$\begin{aligned} W_L^{++} &= -i\frac{4\pi}{k}q^{+a}\bar{q}_a^+, \quad W_R^{++} = -i\frac{4\pi}{k}\bar{q}_a^+q^{+a}, \\ \nabla^{++}q^{+a} &= \nabla^{++}\bar{q}_a^+ = 0. \end{aligned} \quad (4.8)$$

- The $\mathcal{N} = 3$ superfield action, in contrast to the $\mathcal{N} = 0$, $\mathcal{N} = 1$ and $\mathcal{N} = 2$ superfield ABJM actions, *does not involve* any explicit superfield potential, only minimal couplings to the gauge superfields. The correct 6-th order scalar potential emerges *on-shell* after eliminating appropriate auxiliary fields from both the CS and hypermultiplet sectors.
- Three hidden supersymmetries completing the manifest $\mathcal{N} = 3$ supersymmetry to $\mathcal{N} = 6$ are realized by simple transformations

$$\begin{aligned} \delta V_L^{++} &= \frac{8\pi}{k}\epsilon^{\alpha(ab)}\theta_\alpha^0 q_a^+ \bar{q}_b^+, \\ \delta V_R^{++} &= \frac{8\pi}{k}\epsilon^{\alpha(ab)}\theta_\alpha^0 \bar{q}_a^+ q_b^+, \\ \delta q^{+a} &= i\epsilon^{\alpha(ab)}\nabla_\alpha^0 q_b^+, \end{aligned} \quad (4.9)$$

where ∇_α^0 is the properly covariantized derivative with respect to θ_α^0 .

- The hidden R-symmetry transformations extending the R-symmetry of the $\mathcal{N} = 3$ supersymmetry to $SO(6)$ also have a very transparent representation in terms of the basic analytic superfields.
- The $\mathcal{N} = 3$ harmonic superspace formulation makes manifest that the hidden $\mathcal{N} = 6$ supersymmetry is compatible with other product gauge groups, e.g. with $U(N) \times U(M)$, $N \neq M$, and with other types of bi-fundamental representation for the hypermultiplets. The hidden supersymmetry transformations have the universal form in all cases and suggest a simple criterion as to which gauge groups admit this hidden supersymmetry. In this way one can e.g. reproduce, at the $\mathcal{N} = 3$ superfield level, the classification of admissible gauge groups worked out at the component level by Schnabl and Tachikawa in [17].
- The enhancement of the hidden $\mathcal{N} = 6$ supersymmetry to $\mathcal{N} = 8$ and R-symmetry $SO(6)$ to $SO(8)$ in the case of the gauge group $SU(2)_k \times SU(2)_{-k}$ is also very easily seen in the $\mathcal{N} = 3$ superfield formulation. Actually, this enhancement arises already in the case of the gauge group $U(1) \times U(1)$ with a doubled set of hypermultiplets (with **16** physical bosons as compared to **8** such bosons in the “minimal” $U(1) \times U(1)$ case [18]).

5 Outlook

In conclusion, let me list some further problems which can be studied within the $\mathcal{N} = 3$ superfield formulation sketched above.

- Construction and study of the quantum effective action of the ABJM-type models in the $\mathcal{N} = 3$ superfield formulation. The fact that the superfield equations of motion are given solely in the analytic subspace hopefully implies some powerful non-renormalizability theorems [19].
- Computing the correlation functions of composite operators directly in the $\mathcal{N} = 3$ superfield approach as comprehensive checks of the considered version of the AdS_4/CFT_3 correspondence.
- A study of interrelations between the low-energy actions of M2- and D2-branes using the Higgs mechanism [20], in which the second system is interpreted as a Higgs phase of the first one.
- Constructing the full effective actions of M2-branes in terms of the $\mathcal{N} = 3$ superfields (with a Nambu-Goto action for scalar fields in the case of one M2-brane and its nonabelian generalization for N branes).
- ETC ...

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References

- [1] Aharony, O., Gubser, S. S., Maldacena, J. M., Ooguri, H., Oz, Y.: Large N field theories, string theory and gravity, *Phys. Rept.* 323 (2000) 183, [arXiv:hep-th/9905111](#).
- [2] Schwarz, J. H.: Superconformal Chern-Simons theories, *JHEP* 0411 (2004) 078, [arXiv:hep-th/0411077](#).
- [3] Bagger, J., Lambert, N.: Modeling multiple M2’s, *Phys. Rev. D* 75 (2007) 045020, [arXiv:hep-th/0611108](#); Gauge symmetry and supersymmetry of multiple M2-branes, *Phys. Rev. D* 77 (2008) 065008, [arXiv:0711.0955 \[hep-th\]](#); Comments on multiple M2-branes, *JHEP* 0802 (2008) 105, [arXiv:0712.3738 \[hep-th\]](#); Three-algebras and $N = 6$ Chern-Simons gauge theories, *Phys. Rev. D* 79 (2009) 025002, [arXiv:0807.0163 \[hep-th\]](#).
- [4] Gustavsson, A.: Algebraic structures on parallel M2-branes, *Nucl. Phys. B* 811 (2009) 66, [arXiv:0709.1260 \[hep-th\]](#); Selfdual strings and loop space Nahm equa-

- tions, *JHEP* 0804 (2008) 083, [arXiv:0802.3456 \[hep-th\]](#).
- [5] Aharony, O., Bergman, O., Jafferis, D. L., Maldacena, J.: $N = 6$ superconformal Chern-Simons-matter theories, M2-branes and their gravity duals, *JHEP* 0810 (2008) 091, [arXiv:0806.1218 \[hep-th\]](#).
- [6] Benna, M., Klebanov, I., Klose, T., Smedback, M.: Superconformal Chern-Simons Theories and AdS_4/CFT_3 Correspondence, *JHEP* 0809 (2008) 072, [arXiv:0806.1519 \[hep-th\]](#).
- [7] Mauri, A., Petkou, A. C.: An $N = 1$ superfield action for M2 branes, *Phys. Lett. B* 666 (2008) 527, [arXiv:0806.2270 \[hep-th\]](#).
- [8] Cherkis, S., Sämann, C.: Multiple M2-branes and generalized 3-Lie algebras, *Phys. Rev. D* 78 (2008) 066019, [arXiv:0807.0808 \[hep-th\]](#).
- [9] Cederwall, M.: $N = 8$ superfield formulation of the Bagger-Lambert-Gustavsson model, *JHEP* 0809 (2008) 116, [arXiv:0808.3242 \[hep-th\]](#); Superfield actions for $N = 8$ and $N = 6$ conformal theories in three dimensions, *JHEP* 0810 (2008) 070, [arXiv:0809.0318 \[hep-th\]](#).
- [10] Bandos, I. A.: NB BLG model in $N = 8$ superfields, *Phys. Lett. B* 669 (2008) 193, [arXiv:0808.3568 \[hep-th\]](#).
- [11] Samtleben, H., Wimmer, R.: $N = 8$ Superspace Constraints for Three-dimensional Gauge Theories, *JHEP* 1002 (2010) 070, [arXiv:0912.1358 \[hep-th\]](#).
- [12] Zupnik, B. M., Khetselius, D. V.: Three-dimensional extended supersymmetry in harmonic superspace, *Sov. J. Nucl. Phys.* 47 (1988) 730.
- [13] Kao, H.-C., Lee, K.: Self-dual Chern-Simons Higgs systems with $N = 3$ extended supersymmetry, *Phys. Rev. D* 46 (1992) 4691.
- [14] Buchbinder, I. L., Ivanov, E. A., Lechtenfeld, O., Pletnev, N. G., Samsonov, I. B., Zupnik, B. M.: ABJM models in $N = 3$ harmonic superspace, *JHEP* 0903 (2009) 096, [arXiv:0811.4774 \[hep-th\]](#).
- [15] Galperin, A., Ivanov, E., Kalitzin, S., Ogievetsky, V., Sokatchev, E.: Unconstrained $N = 2$ Matter, Yang-Mills and supergravity theories in harmonic superspace, *Class. Quant. Grav.* 1 (1984) 469.
- [16] Galperin, A. S., Ivanov, E. A., Ogievetsky, V. I., Sokatchev, E. S.: *Harmonic Superspace*, Cambridge University Press, 2001, 306 p.
- [17] Schnabl, M., Tachikawa, Y.: *Classification of $N=6$ superconformal theories of ABJM type*, [arXiv:0807.1102 \[hep-th\]](#).
- [18] Bandres, M. A., Lipstein, A. E., Schwarz, J. H.: Studies of the ABJM Theory in a Formulation with Manifest $SU(4)$ R-Symmetry, *JHEP* 0809 (2008) 027, [arXiv:0807.0880 \[hep-th\]](#).
- [19] Buchbinder, I. L., Ivanov, E. A., Lechtenfeld, O., Pletnev, N. G., Samsonov, I. B., Zupnik, B. M.: Quantum $N = 3$, $d = 3$ Chern-Simons Matter Theories in Harmonic Superspace, *JHEP* 0910 (2009) 075, [arXiv:0909.2970 \[hep-th\]](#).
- [20] Mukhi, S., Papageorgakis, C.: M2 to D2, *JHEP* 0805 (2008) 085, [arXiv:0803.3218 \[hep-th\]](#).

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